What Is Studied in, Written For, and Remembered From Western Oregon University's Bridge Course *(Preliminary report)*

Expanded Notes from a Talk Given at the 2008 Joint Mathematics Meetings

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What Is Studied in . . . Western Oregon University's Bridge Course

2006 Survey of Catalogs:
Top 7 Topics in Bridge Courses Nationally

Click Here for Survey Notes or go to http://www.wou.edu/~wardm/BridgeCourseSlidesWinterMeetings2007forWebPage.pdf

Set Theory	83.5%
Logic	78.3%
Functions	55.7%
Relations	54.6%
Methods of proof	40.2%
Induction	33.0%
Equivalence relations	18.6%

2007 JMM Straw Poll Results
Topics "Essential" for a Bridge Course

Topic	Tally
Logic	14
Induction	9
Set Theory	9
Methods of proof	4
Functions & Relations	4
23 other topics	1-2

Introduction to Proof
WOU's Bridge Course *à la* M. Ward

Set Theory	1.5 wks.
Logic (including quantifiers)	2 wks.
Functions & Relations	2 wks.
Methods of proof	3 wks.
Induction	0.5 wk.
Equivalence relations	Indep.
Definitions	0.5 wk.
Writing	All

(Text: *How to Prove It: A Structured Approach 2e*, D. Velleman)

Conclusion

- By those measures, WOU's bridge course (as taught by M. Ward) is totally typical and entirely unexceptional in content. The possible exception is the explicit instruction in the nature and role of mathematical definitions.
- Being an official Writing Intensive course results in some (hopefully) note worthy writing assignments.

What Is . . . Written For . . . Western Oregon University's Bridge Course

Writing Assignment: The Obvious

- Instruction in the conventions of proof writing.
- My personal expectations for the class (low-context writing = write to a dim-witted peer not an informed instructor).
- Selected homework proofs written and rewritten in mathematical prose based on the given conventions and expectations.

Maria Fung's Writing Sequence: Audience

1. Informal Mathematical Writing

   Proof of a simple fact to an untrained, but logical and skeptical, person.

   Your mother has always wondered why the difference of the squares of two consecutive natural numbers is always odd. Write her a one-page letter explaining to her how and why this works. Assume that your mother has not had any formal training in mathematics.

2. Formal Mathematical Writing for a Peer

   Rigorous, but low-context, proof.

   Your friend at Rival University is taking a proofs course. He is very confused about sets and their operations (unions, intersections, complements, difference). He cannot even begin to understand De Morgan's laws. Write a one-to-two page letter to him explaining thoroughly the concepts of sets and their operations. You should definitely include plenty of examples to illustrate your points. Also, you might want to use Venn diagrams to aid your friend's understanding.

3. Formal Mathematical Writing for a Journal

   Rigorous, but higher-context, proof with attention to formatting details.

   Go to the library and find an article in one of the three MAA publications for expository papers . . . Read the article and summarize its most important ideas in one to two paragraphs. Then write up a proof (or several, for extra credit) of the theorem that states the existence of infinitely many primes. Make sure you use journal style formal mathematical writing.

"What is Math?" Sequence:
Writing *about* Mathematics

Part I. Your own opinion.

Informal Writing:
Free-write (for 5 minutes each) all thoughts that come to mind when you think of the questions,

"What is mathematics?"

"Is mathematics good for anything?"

Reporting-out: Student read their free-writing in groups of 3, then each group shares something with the class. This could, but usually does not, produce and interesting class discussion. However, it does establish a baseline.

Click here for the full assignment. or go to http://www.wou.edu/~wardm/JMM2008Talk/280WhatIsMath1F06.pdf

Part II. Others' opinions.

Readings:

- Chapters 2 and 3, pages 13–45, of *Nature's Numbers* by Ian Stewart.
- "What is Mathematics?" (pp. 6-8) and sections 1-4 of "Utility" (pp. 79-87) in *The Mathematical Experience* by Davis and Hersh.
- The first part of "Entering Mathematical Sciences at the Graduate Level" by Robert V. Moody.
- "The Allegory of the Epiphany at the Fountain ...tell me THIS isn't crazy" by Car Talk's Tom Magliozzi and the associated feedback (online).

Formal Writing:
Two essays, "What is math?" and "Is math good for anything?" summarizing and comparing the views of the authors of the readings.

Click here for the full assignment. or go to http://www.wou.edu/~wardm/JMM2008Talk/280WhatIsMath2F06.pdf

Click here for grading rubric. or go to http://www.wou.edu/~wardm/JMM2008Talk/WhatIsMathPt2rubric.pdf

Part III. A look at research mathematics.

Find an issue of *The American Journal of Mathematics* or *Transactions of the American Mathematical Society*.

Look at each of them for about 5 minutes, reading the introductions and looking at the *structure* of the articles.

Formal Writing:
An entertaining essay on what you saw in the articles and your reaction to it. (More detailed instructions given.)

Goals: To show that math is communicated in prose and that math has new creations/discoveries just like other arts and sciences.

Click here for full assignment. or go to http://www.wou.edu/~wardm/JMM2008Talk/280JournalReadingS07.pdf

Part IV. "The Proof" – a research mathematician at work.

Watch the "The Proof," an episode of "Nova" on the proof of Fermat's Last Theorem

Informal Writing:
During the video, write down any questions that come to mind and note anything that you find interesting.

After the video, free-write your reaction to it and comments about it.

Free-write your additional thoughts, changes of opinion, or whatever regarding questions "What is math?" and "Is math good for anything?"

Report-out as with original "What is math?" free-write.

Click here for full assignment. or go to http://www.wou.edu/~wardm/JMM2008Talk/280TheProofS07.pdf

Part V. Conclusion–your own opinion again

Formulate your own concise and well reasoned answers to the questions "What is mathematics?" and "Is mathematics good for anything?"

Writing Assignments: Writing to *Learn* about Mathematics

Part I. Definitions

Reading: "Surprises from Mathematics Education Research: Student (Mis)use of Mathematical Definitions," *American Mathematical Monthly* Volume 111, Number 5, May 2004, pp. 411-424 (available in the periodical section of the library or via a link at Mike Ward's webpage `http://www.wou.edu/~wardm/ward.html`), read sections 1, 2 and 4 (Introduction, Framework, and The Surprises).

Writing: A structured essay on the key points about mathematical definitions.

Click here for full assignment. or go to `http://www.wou.edu/~wardm/JMM2008Talk/DefinitionsF07.pdf`

Part II. Equivalence Relations

Reading: The sections in the text about equivalence relations.

Writing: Without any formal instruction in class, each student writes a structured essay on equivalence relations that includes the formal definition, the student's intuition about equivalence relations, original examples and non-examples, and a problem about equivalence classes.

Click here for full assignment. or go to `http://www.wou.edu/~wardm/JMM2008Talk/EquivClassProjectF07.pdf`

What Is . . . Remembered From Western Oregon University's Bridge Course

Fall 2007 Student Survey Goals

Public: What do our students remember and actually use from our bridge course? What do they think is important and does that change with experience?

Private: Is the list of topics remembered and consciously used so short that it might readily be incorporated into an existing content course, eliminating the need for the bridge course? Or is the list so long that I could quit fretting about the value of our bridge course?

Fall 2007 Survey Instruments

Week 1: Without referring to any notes or texts, list what you remember from Introduction to Proof. In your opinion at this time, which of the above are the most important? (Mark them with a *)

Weeks 2-6: Between now and the due date, please note each use of something from Introduction to Proof and indicate whether
(A) you consciously thought "I learned something in proofs class that will help me here" while working or
(B) you did something and only later realized it was covered in the proofs class or
(C) you were shown how to do something and then realized it was from proofs class.

Week 7: Same as Week 1.

Data

Wk. 1: Top 10 Most Frequently Remembered
(ordered according to reported importance)

Weeks 2-6: Top 10 Most Frequently Used
(That is, most frequently reported!)
(ordered by the number of conscious uses)

Wk. 7: Top 10 Most Frequently Remembered
(ordered according to reported importance)

Week 1 Memories	Weeks 2-6 Usage	Week 7 Memories
contrapositive	contradiction	contradiction
existential generalization	set equality	existential general.
universal generalization	statement negation	universal general.
"if-then strategy"	quantifiers	"if-then strategy"
truth tables	"if-then strategy"	induction
contradiction	use definitions	quantifiers
set notation	contrapositive	set equality
statement negation	induction	contrapositive
unnamed strategies	unnamed strategies	negation
induction	set notation	use definitions

Observations

- 7 of 10 Week 1 Memories are methods of proof. Good! That's what the course aims to teach.
- Note the curious persistence of existential and universal generalization as both remembered and important despite the absence of reported usage.
- The emergence of definitions was gratifying, since explicit instruction about definitions was the only thing that set WOU's bridge course apart from the average.

Conclusions

1. A casual survey does not produce very convincing results. The week 2-6 usage logs seemed highly suspect. More serious student buy-in or an impartial observer is necessary.

2. Students report remembering and using a number of key topics from their bridge course.

3. Taken at face value, the data suggest that our bridge course may be worthwhile. (But it's not convincing enough to end my fretting.)