

## GS104 Basics Review of Math

### I. MATHEMATICS REVIEW

#### A. Decimal Fractions, basics and definitions

1. Decimal Fractions - a fraction whose denominator is 10 or some multiple of 10 such as 100, 1000, 10000, etc.

$$8/10 = 0.8 \quad 79/100 = 0.79 \quad 183/1000 = 0.183$$

$$5925/10000 = 0.5925$$

1st place to right of decimal = tenths

2nd place to right of decimal = hundredths

3rd place to right of decimal = thousandths

4th place to right of decimal = 10 thousandths

5th place to right of decimal = 100 thousandths

6th place to right of decimal = millionths

Number	Powers of 10	Exponential Form
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$$1,000,000 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^6$$

$$100,000 = 10 \times 10 \times 10 \times 10 \times 10 = 10^5$$

$$10,000 = 10 \times 10 \times 10 \times 10 = 10^4$$

$$1000 = 10 \times 10 \times 10 = 10^3$$

$$100 = 10 \times 10 = 10^2$$

$$10 = 10 = 10^1$$

$$1 = 1 = 10^0$$

$$0.1 = 1/10 = 10^{-1}$$

$$0.01 = 1/10 \times 1/10 = 10^{-2}$$

$$0.001 = 1/10 \times 1/10 \times 1/10 = 10^{-3}$$

$$0.0001 = 1/10 \times 1/10 \times 1/10 \times 1/10 = 10^{-4}$$

$$0.00001 = 1/10 \times 1/10 \times 1/10 \times 1/10 \times 1/10 = 10^{-5}$$

#### B. THE METRIC SYSTEM AND CONVERSION

1. Metric system- developed in Europe (France) in 1700's, offered as an alternative to the British or English system of measurement.
2. S.I./metric system involves measurements of length (meter), mass or weight (kilogram), temperature (celsius), time (second), and volume (litre).
3. Metric system based on powers of 10 and a decimal approach with prefixes attached to the basic units of measurement to indicate the power of 10 in question.

Greek prefixes > 1 base unit, Latin prefixes < 1 base unit

- |                       |  |
|-----------------------|--|
| 1. Mega = $10^6$      | e.g. 1 megameter = $1 \times 10^6$ meter |
| 2. Kilo = $10^3$      | 1 kilometer = $1 \times 10^3$ meters     |
| 3. Hecto = $10^2$     | 1 Hectometer = $1 \times 10^2$ meters    |
| 4. Deka = $10^1$      | 1 Dekameter = $1 \times 10^1$ meters     |
| 5. Base unit = $10^0$ | 1 meter = $1 \times 10^0$ meters         |
| 6. Deci = $10^{-1}$   |  |
| 7. Centi = $10^{-2}$  | and so on                                |
| 8. Milli = $10^{-3}$  |  |
| 9. Micro = $10^{-6}$  |  |
| 10. Nanno = $10^{-9}$ |  |
| 11. Pica = $10^{-12}$ |  |

The movement of the decimal point to the left or right of the given quantity of a unit is all that is needed to change a given type of unit to the next higher or lower unit:

e.g.  $1 \text{ m} = 10 \text{ dm} = 100 \text{ cm} = 1000 \text{ mm} = 1,000,000 \text{ }\mu\text{m}$   
 $1 \text{ m} = 0.1 \text{ Dam} = 0.01 \text{ Hm} = 0.001 \text{ Km} = 0.0000001 \text{ Mm}$

#### 4. METRIC MEASUREMENT OF DISTANCE

a. Based on the meter (analogous to the yard in English system)

$1 \text{ Km} = 1000 \text{ m}$ ,  $1 \text{ Hm} = 100 \text{ m}$ ,  $1 \text{ Dam} = 10 \text{ m}$ ,  $1 \text{ m} = 1 \text{ m}$ ,  
 $1 \text{ dm} = 0.1 \text{ m}$ ,  $1 \text{ cm} = 0.01 \text{ m}$ ,  $1 \text{ mm} = 0.001 \text{ m}$ ,  $1 \text{ }\mu\text{m} =$   
 $0.000001 \text{ m}$

b. Conversion of One metric unit to another

e.g. convert 8.9 km to m:  $8.9 \text{ km} \frac{1000 \text{ m}}{1 \text{ km}} = 8900 \text{ m}$

e.g. convert 1230 m to km:  $1230 \text{ m} \frac{1 \text{ km}}{1000 \text{ m}} = 1.23 \text{ km}$

#### 5. METRICATION OF AREA (length x length)

a. SI units:  $\text{km}^2$ ,  $\text{m}^2$ ,  $\text{cm}^2$ , etc.

b. Metric equivalent of Acre = Hectare (Ha) =  $100 \text{ m} \times 100 \text{ m}$   
which equals  $10,000 \text{ m}^2$ ; i.e.  $10,000 \text{ m}^2/\text{Ha}$

e.g. determine the no. of hectares in a plot of land:  $1.6 \text{ km} \times 1.2 \text{ km} = 1600 \text{ m} \times 1200 \text{ m} =$   
 $1,920,000 \text{ m}^2$  ( $1 \text{ Ha}/10,000\text{m}^2$ )= 192 Ha

#### 6. METRICATION OF VOLUME (length x length x length)

a. volume- the amount of space within a container or enclosed within a solid

- b. SI units of volume: cubic meters which can be equated to litres.
- c. Can use same metric-prefix approach as given for meters, can be used with litres as well

e.g. 1 l = 1000 ml = .001 kl and so on

e.g. convert 17 litres to milliliters:

$$17 \text{ l (1000 ml/l)} = 17,000 \text{ ml}$$

- d. E.g. of problems converting volume in metric system

- (1) Find the volume in liters of a rectangular tank (l<sub>x</sub>w<sub>x</sub>h) 2 m x 20 dm x 28 cm

## 7. METRICATION OF MASS

- a. Mass - quantity of material contained in a given body

- (1) Weight - measure of the force of gravity upon a given body.

Thus mass and weight are interchangeable under a given force of gravity, but may differ in cases of 2 different gravitational forces (e.g. a given mass will have different weights on the earth as compared to the moon ( $G_{\text{moon}} = 1/6 G_{\text{earth}}$ ), but the mass or quantity of material occupying space will be same on earth as on the moon).

- b. Metric unit of measuring mass = gram, kilogram, etc.

- (1) converting from volume to capacity to weight:

1000 cu. cm = 1000 ml = 1000 gram of pure water

For pure water: 1 L = 1 Kg, thus 1 gm of water = 1 ml of water = 1 cu. cm

- c. E.g. of metric conversions: convert 2700 mg to grams

2700 mg (1 gm/1000 mg) = 2.7 grams

## 8. METRIC MEASUREMENT OF TEMPERATURE

- a. Metric unit = celsius, English unit = Farenheit

- b. water freezes at 32° F = 0° C water boils at 212° F = 100° C

- c. Conversion Factors:

- (1) From C to F:  $F = 9/5C + 32^\circ$

- (2) From F to C:  $C = 5/9(F - 32^\circ)$

- (a) E.g. convert 40 C to F  
 $F = 9/5(40) + 32 = 104^\circ \text{ F}$

- d. CONVERSION FROM ENGLISH SYSTEM TO METRIC AND VICE VERSA

- (1) Conversion charts/factors given for units of length, area, volume, and weight/mass on p. 300.
- (2)
  - (a) E.g. of conversion problems:
  - (b)

Given that 1 yard = 0.9144 m, how many meters are there in 100 yards?  
 100 yd (0.9144 m/1 yd) = 91.44 m.

II. Algebra Review / Graph Function Review

A. Unit Conversion and Unit Management

- 1. Keeping track of unit dimensions in equations is very important
- 2. Unit algebra is based on simple unit cancelling

E.g. Given the fractional equation:  $4 * \frac{2}{4}$  (note here "\*" = times)

since there is a 4 in the numerator and 4 in the denominator, we can short-cut by simply cancelling out the 4 above, and 4 below ( $4/4 = 1$ )... and we find that the equation is equal to 2.

By analogy, given the algebraic equation:  $Y * \frac{2}{Y}$  (note here "\*" = times)

since there is a "Y" in the numerator and Y in the denominator, we can short-cut by simply cancelling out the Y above, and Y below ( $Y/Y = 1$ ).... and we find that the equation is equal to 2.

By analogy, given that 1 mile = 5280 ft, we can convert 20,000 ft to miles by using unit algebra:

- 1) set up the equation so that the units you are trying to cancel are in the numerator and denominator
- 2) check to see if the end unit is the one you're looking for....

$20,000 \text{ ft} * \frac{1 \text{ mile}}{5280 \text{ ft}} = 3.79 \text{ miles..}$  in this case the ft / ft cancels, leaving miles as the unit

EXAMPLE TO TRY: given that 1 in = 2.54 cm, 1 ft = 12 in, and 1 mi = 5280 ft; How many centimeters are in 863 ft? Remember you are going from ft to cm, manage your units so that all cancel, except cm!

B. Algebraic Manipulation of Exponents

1. Negative Exponents

$$a^{-n} = 1/a^n$$

2. The zero power (any no. raised to the zero power = 1)

$$a^0 = 1$$

3. Power of one (any no. raised to the 1st power = that number)

$$a^1 = a$$

4. Multiplication (exponential nos. with the same base)

$$a^m * a^n = a^{m+n}$$

5. Division

$$a^m/a^n = a^{m-n}$$

6. Distribution

$$(a*b)^n = a^n*b^n$$

$$(a^m)^n = a^{m*n}$$

C. Dividing Fractions

1. When dividing by a fraction, invert the fraction and multiply

e.g.  $1/(1/4) = 1 * (4/1) = 4$

e.g.  $(m/sec)/sec = (m/sec)*(1/sec) = m/sec^2$

D. Graphing Review

1. Axis

a. Y axis = vertical axis (ordinate)

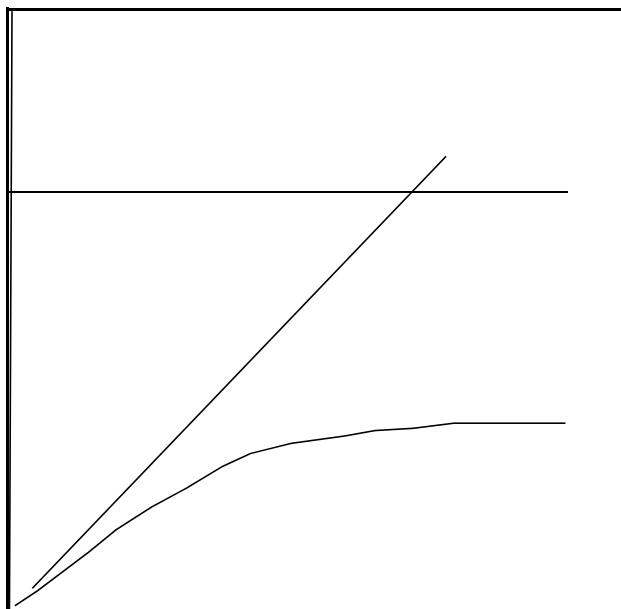
b. X axis = horizontal axis (abscissa)

2. Graph Trends (see attached figures)

a. Linear Increase / Decrease

b. Constant

c. Parabolic (curvilinear) Increase / Decrease



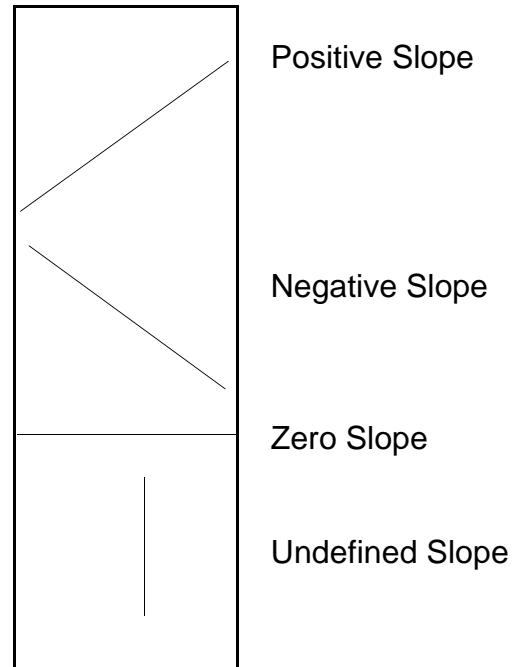
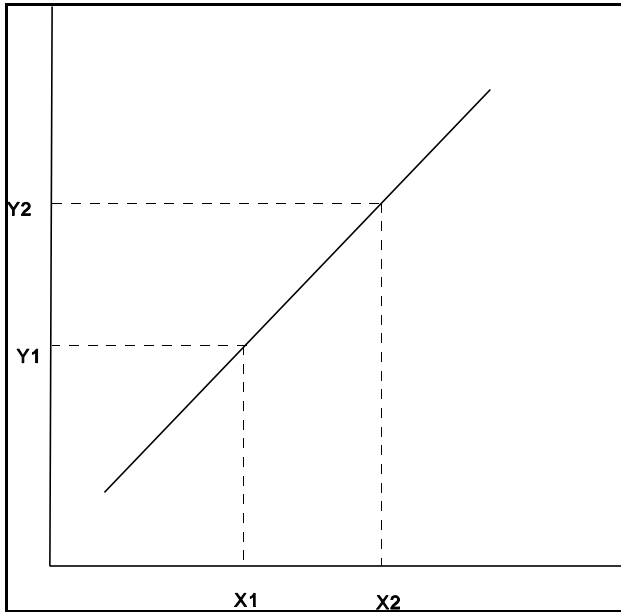
Linear Increase

Constant

Parabolic

### 3. Determining Slopes of Lines

a. slope of any line on a graph = rise / run =  $(Y_2 - Y_1) / (X_2 - X_1)$



#### E. Rearranging equations algebraically

1. By using simple algebra, equations can be re-arranged to solve for other unknowns:
2. Examples

Given velocity and time, how to figure distance traveled during the time period?

Velocity  $V = d/t$  rearranged to... multiply both sides of equation by t...  $d = V*t$

Given velocity and distance, how to figure time of travel?

Velocity  $V=d/t$  rearranged to...  $t = d/V$

Given acceleration and time, how to figure velocity acquired during the time period?

Acceleration  $A = V/t$  rearranged to... multiply both sides of equation by t...  $V = A*t$

Example: you are driving a constant 50 km / hr for 35 minutes, how far have you traveled?

Example: you are accelerating in your car at 10 km/sec/sec for 90 sec, what is your velocity?

## Common Conversion Factors

### *Time*

1 b.y. = 1,000,000,000 years  
1 m.y. = 1,000,000 years  
1 year = 365 days  
1 day = 24 hours  
1 hour = 60 minutes  
1 minute = 60 seconds

### *Length*

1 mile = 5280 feet  
1 foot = 12 inches  
1 yard = 3 feet  
1 inch = 2.54 cm  
1 meter = 3.28 feet  
1 meter = 100 cm  
1 meter = 1000 mm  
1 km = 1000 m  
1 mile = 1.61 km  
1 km = 0.62 miles

### *Mass / Weight*

1 pound = 16 ounces  
1 ton = 2000 pounds  
1 kg = 1000 gm  
1 ounce = 28 gm  
1 kg = 2.2 pounds

### *Volume*

1 gallon = 4 quarts  
1 quart = 0.95 litres  
1 litre = 1.05 quarts  
1 litre = 1000 ml  
1 ml = 1 cubic cm (of pure water)