HYDRAULICS APPENDIX

1 INTRODUCTION

Relevant to the discussion of habitat restoration, hydraulics can be defined as the laws governing the movement of water within a channel and the forces generated by this movement. Hydraulic effects result in movement of sediment, erosion of channel banks and scour of the channel bed. These processes help form, as well as respond to, the geomorphic conditions of the stream. This appendix describes how to calculate hydraulic conditions at a section and discusses one-dimensional models for characterizing flow along a reach, shear stress (erosive forces along a bank or bed) and scour depth in stream channels.

Although the material presented in this appendix is intended for engineers experienced with hydraulics, it may be beneficial reading for anyone involved with stream restoration. Readers unfamiliar with basic hydraulics are referred to the "Recommended Reading" section at the end of the appendix. Various methods for sizing riprap are listed in the Riprap technique of the <u>Integrated Streambank Protection Guidelines</u> (ISPG). Channel bed design is described in the Stream Simulation Culvert design of the Fish Passage at Road Crossing guidelines.

2 MANNING'S EQUATION

Manning's Equation is probably the most commonly used formula for basic hydraulic calculation in natural channels. In its most basic form, the equation relates flow velocity to hydraulic radius, hydraulic roughness and channel slope. Using Manning's Equation in its various forms, one can determine:

- Average water velocity given cross-sectional geometry, depth, slope, and roughness;
- Channel discharge given cross-sectional geometry, depth, slope, and roughness;
- Channel roughness given cross-sectional geometry, slope, depth, and discharge;
- Channel slope given cross-sectional geometry, discharge, depth, and roughness; and
- Channel depth given cross-sectional geometry, discharge, slope, and roughness.

Manning's equation assumes steady and uniform flow. When the velocity at any given point remains constant with respect to time, then a flow is considered *steady*. If flow depth does not change with location along the channel, then the flow is *uniform*. In reality, steady and uniform flow is practically nonexistent in natural settings. Nonetheless, Manning's equation is commonly used as a relatively simple and convenient tool for hydraulic analysis of natural streams. It is generally understood and accepted that the results are approximate, and designers should keep this in mind when applying its results.

Manning's Equation can be written in either velocity or discharge terms as follows:

$$V = (1.49/n)(R_{h}^{2/3} S_{e}^{1/2})$$
(Equation 1)

$$Q = (1.49/n)(A R_{h}^{2/3} S_{e}^{1/2})$$
(Equation 2)
Where: V = average cross-sectional velocity (ft/sec)
n = Manning's roughness value
Q = discharge (cubic ft/sec)
S_{e} = energy slope in (ft/ft)
R_{h} = hydraulic radius (ft) = A/P
Where: A = cross-sectional area of flow (ft²)
P = wetted perimeter (ft)

The Manning's roughness value *n* accounts for the resistance to flow presented by the channel. Higher *n* values correspond to *rougher* channels, such as those formed by large rock, wood, and rigid vegetation. Lower *n* values correspond to channels with smoother boundary materials and lower sinuosity. The Manning's roughness value also varies with stream stage, as boundary materials such as boulders have a higher relative roughness at low stream stage than at higher stages.

Appropriate values for *n* are typically estimated based on tables for *n* developed through empirical study. Methods for calculating *n* are also available. Guidance for determining appropriate n values can be found in most hydraulic analysis/design references including Chow $(1959)^{1}$ and others^{2,3,4}.

2.1 Continuity

The modification of Manning's equation from the form shown in equation (1) to that shown in equation (2) is based on the fundamental relation:

Q = VA (Equation 3)

Where: Q = discharge (cubic ft/sec) V = average cross-sectional velocity (ft/sec)A = cross-sectional area of flow (ft²)

Assuming the cross-sectional area of flow "A" is measured normal to the flow direction, the relation expressed by equation (3) holds true for any cross-section on a stream. If discharge is constant throughout a stream reach, then the flow is considered to be *continuous*, and the following relation is true (Chow, 1959).

 $Q = V_1 A_1 = V_2 A_2 = \dots$ (Equation 4)

Where the subscripts denote different locations within the reach (Chow, 1959). Equation (4) is commonly known as the *continuity equation*. The continuity equation holds true as long as discharge within the reach is constant (there is no additional water flowing into or out of the reach).

2.2 Modeling Backwater Effects

With the exception of long uniform stream reaches, flow along a stream is often controlled by downstream (during subcritical flow regime) or upstream (supercritical flow regime) channel conditions. Backwatering occurs when a downstream flow control such as a constriction, higher bed elevation (e.g. crest of riffle downstream of a pool) or increased roughness creates a ponding effect that forces a higher water surface elevation at an upstream section then would occur in a freely draining condition. The Manning's equation is not capable of modeling backwater effects caused by downstream flow control. One-dimensional models are necessary to quantify backwater effects. One-dimensional numerical surface water models are based on continuity and momentum and they assume cross-section averaged flow, in steady state conditions, and a flat water surface elevation across each cross section.

Examples of these models include:

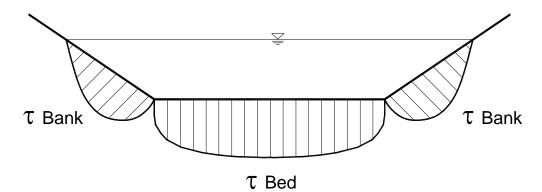
- 1. US Army Corps of Engineers (USACE), Hydrologic Engineering Center, River Analysis System Model, (HEC-RAS). HEC-RAS analyzes networks of natural and man-made channels and computes water surface profiles for subcritical or supercritical flow based on steady or unsteady one-dimensional flow hydraulics. The system can handle a full network of channels, a dendritic system, or a single river reach with an analysis of all types of hydraulic structures. Flow distributions between channels (such as around an island) can be modeled with HEC-RAS version 3.0 using the flow split optimization option.
- Federal Highway Administration (FHWA), Water Surface Profile Model (WSPRO). WSPRO computes water surface profiles for subcritical, critical, or supercritical flow as long as the flow can be reasonably classified as one-dimensional, gradually varied, and steady flow. It can be used to analyze open-channel flow, flow through bridges (single or multiple openings), embankment overflow, floodway analysis, and bridge scour.

Occasionally projects will require predictions of complex flow patterns warranting the use of two-dimensional hydraulic models. Examples of such projects include: wide shallow streams, bays, estuaries or braided streams or when secondary currents are important such as around bends or groins. Two-dimensional models in use include Finite Element Surface-Water Modeling System (FESWMS) from FHWA and RMA-2 (USACE).

3 SHEAR STRESS

Shear stress is an important parameter in habitat restoration design, because all materials, whether manufactured or natural, used for habitat restoration must be able to withstand the expected shear stress at the design discharge. Thus, in design, all materials and vegetation types are chosen based on the expected shear for a given flow (for example, the 50-year discharge) at their point of installation. Shear stress is typically measured in units of pounds per square foot (psf).

On any given bank, the material and vegetation types required to resist erosion may vary with location. Lane's diagram, **Hydraulics Figure 1**, shows theoretical distribution of shear stress on streambed and banks on a straight section of trapezoidal channel. Based on Lane's diagram, materials and plants of greater shear resistance are required lower on the bank, while a lighterduty treatment may be sufficient near the top of the bank. When designing habitat restoration features that include temporary surface protection such as biodegradable fabric, the designer must be sure that the shear resistance of both the temporary protection (e.g., coir fabric) and the long term surface treatment (vegetation) is adequate to withstand hydraulic forces at that location. In addition, when designing using vegetation as the primary erosion protection, factors such as species, site aspect, shade, soil type, moisture conditions, and local climate must all be considered.



Typical shear stress distribution in a channel.

Hydraulics Figure 1. Theoretical distribution of shear stress on bed and banks.

Typical permissible shear stresses for various materials are shown in **Hydraulics Table 1**. As can be seen in the table, the range of materials for which such information is available is limited. Often, the information listed in **Hydraulics Table 1** must be extrapolated or used merely as an aid in estimating the shear resistance of similar plants and materials that do not appear there. In addition, there is no standardized testing procedure that accounts for the effects of weather, repetitive inundation, and long-duration inundation. Therefore, the values in **Hydraulics Table 1** should be applied using professional judgment and considering site variables of the project location. Fischenich⁵ provides additional information in Stability Thresholds for Stream Restoration Materials.

Hydraulics Table 1.

Permissible shear stresses of various materials.

Material	Permissible shear stress (psf)
Straw with net	1.4
Coir mats and fabrics	Approx. 1-3 (varies by product)
Synthetic mats	Approx. 2-8 (varies by product)
Class A vegetation	3.7
Weeping lovegrass: excellent stand, average height 30"	
Yellow Bluestem Ischaemum: excellent stand, average height 36"	
Class B vegetation	2.1
Kudzu: dense or very dense growth, uncut	
Bermuda grass: good stand, average height 12"	
Native grass mix (long and short midwest grasses): good stand,	
unmowed	
Weeping lovegrass: good stand, average height 13"	
Lespedeza sericea: good stand, not woody, average height 19"	
Alfalfa: good stand, uncut, average height 11"	
Blue gamma: good stand, uncut, average height 13"	
Class C vegetation	1.0
Crabgrass: fair stand, uncut (10" – 48")	
Bermuda grass: good stand, mowed, average height 6"	
Common lespedeza: good stand, uncut, average height 11"	
Grass-legume mix: good stand, uncut $(6" - 8")$	
Centipedegrass: very dense cover, average height 6"	
Kentucky bluegrass: good stand $(6'' - 12'')$	
Class D vegetation	0.6
Bermuda grass: good stand, cut to 2.5-inch height	
Common lespedeza: excellent stand, uncut (average height 4.5")	
Buffalo grass: good stand, uncut $(3'' - 6'')$	
Grass-legume mix: good stand, uncut $(4'' - 5'')$	
Lespedeza sericea: very good stand cut to 2-inch height	
Class E vegetation	0.4
Bermuda grass: good stand, cut to 1.5-inch height	
Bermuda grass: burned stubble	
1-inch gravel	0.3
2-inch gravel	0.7
6-inch rock riprap	2.0
12-inch rock riprap	4.0

Source: All but "coir mats and fabrics" and "synthetic mats" are from USDOT, 1988.⁶

3.1 Estimating Shear Stress

Shear equations presented in this appendix allow the designer to estimate bed and bank shear in straight stream reaches and bends. In addition, a means of estimating shear as a function of height in the water column is presented. It is assumed that persons utilizing the equations presented in this appendix are well versed in hydraulic analysis and familiar with the concepts of shear and scour. It is recommended that hydraulic analyses be carried out only by a qualified hydraulic engineer or someone with equivalent experience.

3.1.1 Bed Shear Stress in a Straight Reach

Shear stress on the bed is ⁶ :	$\tau_{bed} = \gamma \; S_e \; \; R_h$	(Equation 5)
	γ = the specific weight of water = 62.4 lbs/ft ³ ,	
Therefore:	$\tau_{bed} = 62.4~S_e~R_h$	(Equation 6)
Where:	τ_{bed} = maximum bed shear stress in lb/ft ² (psf) R _h = hydraulic radius in ft. (see below) S _e = energy slope in ft/ft (see below)	

 S_e is the slope of the energy grade line. This slope is usually similar to the hydraulic grade line (water surface) and bed slope (gradient) and is typically replaced by bed slope in hand calculations. By definition, the slopes are equal for steady, uniform flow. A standard and appropriate way to calculate channel slope from a surveyed profile is to base the elevation change on the elevations of the thalweg at "zero flow" points. Zero flow points are the points in the bed that would control the pools upstream of major riffles if there were no water flowing in the channel. They are the low points at the head of riffles. In a braided channel, or channels without defined riffles, the mean bed elevation should be used. The mean bed elevation should be determined from several closely spaced cross-sections. The U.S. Army Corp of Engineers hydraulic program, HEC-RAS, can output bed shear stress as well as energy slope.

 R_h is the hydraulic radius, which is the cross-sectional area of the wetted channel (A) divided by the length of the wetted channel perimeter (P), at the design flow being considered. This value is occasionally replaced by depth of flow, y, but this should only be done when the width of the channel far exceeds the depth of the channel. HEC-RAS will always correctly use A/P. As a rule of thumb, always use $R_h = A/P$.

A common application of the equation is for maximum bed shear stress is:

Maximum bed shear stress in a straight reach: $\tau_{bed} = 62.4$ (S_e) (A/P) (Equation 7)

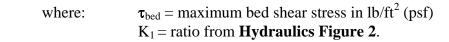
where: $\tau_{bed} = maximum bed shear stress in lb/ft^2 (psf)$ $A = cross-sectional area of flow (ft^2)$ P = wetted perimeter (ft) $S_e = energy slope in (ft/ft)$

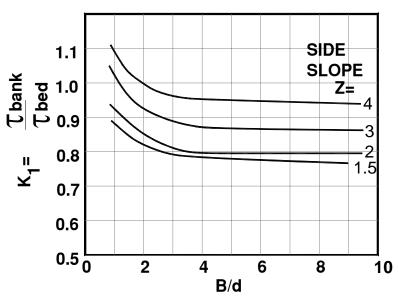
This calculation gives a quantitative measure of the erosive force acting on the bed of the channel.

3.1.1.1 Bank Shear Stress in a Straight Reach

By approximating the channel cross-section as a trapezoid or rectangle, the maximum bed shear stress can be used to estimate the maximum bank shear stress. This stress acts approximately one-third of the distance up the bank (from the bed) and can be approximated by multiplying the maximum bed shear stress by a factor (see Lane's Diagram, **Hydraulics Figure 1**). This factor, K_1 , varies based on channel side slope and the ratio of bottom width to depth as shown in **Hydraulics Figure 2**. This approximation applies only to a relatively straight reach of stream.

Maximum bank shear stress in a straight reach⁶. $\tau_{bank} = K_1 \tau_{bed}$ (Equation 8)





Hydraulics Figure 2. Side slopes, depth/width ratio⁶.

Shear stress on the upper bank can be estimated using Lane's Diagram shown in **Hydraulics Figure 1**. Based on this diagram, side shear vs. depth can be estimated using the following equation:

 $\boldsymbol{\tau}_{x\,=}\ C\,\boldsymbol{\tau}_{bed}$

(Equation 9)

where: $\tau_x =$ bank shear at distance X from stream bottom (psf) $\tau_{bed} =$ maximum bed shear stress (psf) C = coefficient from **Hydraulics Table 2** y = stream depth (ft)

Hydraulics Table 2. Coefficient "C" vs. depth

Distance X	С	С
(feet from stream bottom)	(From Lanes)	(Recommend
		for design)
у	0.0	0.0
0.9 y	0.14	0.14
0.8 y	0.27	0.27
0.67 y	0.41	0.41
0.6 y	0.54	0.54
0.5 y	0.68	0.68
0.4 y	0.79	0.79
0.33 y	0.8	0.8
0.2 y	0.7	0.8
0.1 y	0.5	0.8
0.0 y	0.0	0.8

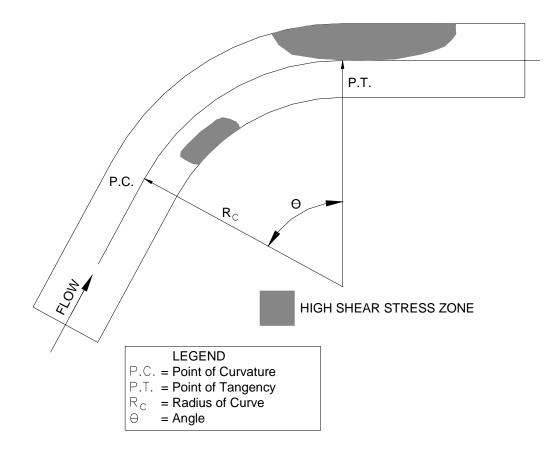
Note: Although Lane's shear diagram indicates zero shear at the base of the bank, for design purposes it is recommended that the maximum bank shear, as calculated above, be assumed to be present for the entire lower 1/3 of the bank height.

3.1.1.2 Shear Stress in Bends⁶

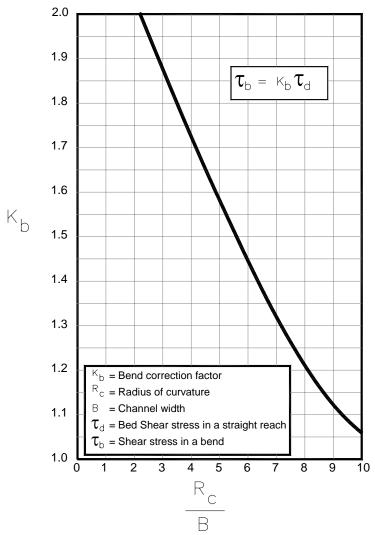
Flow around bends creates secondary currents that exert higher shear forces on the channel bed and banks than those found in straight sections. Several techniques are available for estimating shear stress in bends. A relatively simple and widely used method estimates maximum shear stress on channel banks and bed in bends (this equation does not differentiate between bank and bed shear stress). The maximum bed/bank shear stress in a bend is:

$\tau_{bend} = K$	$\mathbf{t}_{bed} \tag{Equation 10}$
where:	τ_{bend} = maximum shear stress on bank and bed in a bend (psf) τ_{bed} = maximum bed shear stress in adjacent straight reach (psf)
and:	$K_b =$ bend coefficient (dimensionless) $K_b = 2.4 e^{-0.0852(Rc/b)}$ (alternatively, K_b can be determined from Hydraulics Figure 4)
1	

where:	R_c = radius of curvature of bend (ft)
	b = bottom width of channel at bend (ft)



Hydraulics Figure 3. Shear stress distribution in a channel bend.



Hydraulics Figure 4. Bend scour correction factor chart.

The maximum bed/bank shear stress is primarily focused on the bank and bed on the outside portion of the bend (**Hydraulics Figure 3**).

Analysis of the vertical distribution of shear stress on banks in bends is not well defined. Secondary currents found in bends complicate shear analysis in these regions. Equation (8) can be used as a rough estimate of shear distribution on banks in bends, but it does not account for secondary currents. It is recommended that vertical shear distribution in bends be estimated by using Equation (8), judgment based on the severity of the bend and the degree of expected super-elevation of the water surface around the bend. Super-elevation of the water surface around a bend can be estimated as described in the following paragraph.

The water surface elevation increases around the outside of bends as the channel banks exert centrifugal forces on the flow. This super-elevation can be estimated using the following equation:

$\Delta y = V^2 W / (g R_c)$		(Equation 11)
where:	Δy = super-elevation of water surface (ft) V = average velocity of flow (ft/s) W = channel top width (ft) g = acceleration due to gravity (32.2 ft/s ²) R _c = radius of curvature of bend (ft)	

3.2 Scour

Scour is an essential contributor to the creation of fish habitat and its maintenance. Many fishenhancement projects promote scour. It is not the extent or magnitude of the scour that promotes the best habitat, but the frequency of the scour activity. Sites absent of scour tend to provide less habitat complexity than areas subject to moderately frequent scour events, given that intermediate-level disturbances promote aquatic diversity^{7,8}. Sites that are subject to very frequent scour have less habitat value than areas subject to moderately frequent scour events.⁹

This appendix summarizes calculation methods to predict the depths of scour at embankments and instream structures. Accurate prediction of scour depth is invaluable when designing stream bank toes, cross-channel structures such as check dams, and anchoring systems. In addition, the calculation of scour depth allows the designer to predict the effectiveness of instream structures intended to induce scour.

Most of the scour equations presented here were developed to predict hydraulics phenomenon associated with man-made structures, such as bridges, located within relatively large, often sandbed, streams. In general, equations predicting scour in streambeds consisting of gravel and larger material are not considered as reliable as the more widely used equations based on homogeneous fine-grained sand substrate. Because of the lack of widely-used scour equations developed specifically for use on gravel-bed streams, the equations developed for sand-bed streams are presented in this appendix along with methods of modification and interpretation that allow their application to gravel-bed streams with larger bed material.

Paraphrasing passages from Pemberton and Lara¹⁰ on channel scour:

"The design of any structure located either along the riverbank and floodplain or across a channel requires a river study to determine the response of the riverbed and banks to large floods. Knowledge of fluvial morphology combined with field experience is important in both the collection of adequate field data and selection of appropriate studies for predicting the erosion potential.

It should be recognized that many equations are empirically developed from experimental studies. Some are regime-type based on practical conditions and considerable experience and judgment. Because of the complexity of scouring action, it is difficult to prescribe a direct procedure. Bureau of Reclamation practice is to compute scour by several methods and utilize judgment in averaging the results or selection of the most applicable procedures."

3.2.1 Calculating Potential Depth of Scour

Anticipating the maximum scour depth at a site is critical to the design of a bank treatments and structures by defining the type and depth of foundation needed. Scour depth is also useful when designing anchoring systems or estimating the depths of scour pools adjacent to in-channel structures. Determining the maximum depth of scour is accomplished by:

- 1. Applying calculations based on information derived from a complete hydrologic and hydraulic evaluation of the stream.
- 2. Identifying the type(s) of scour expected. (See next section, Types of Scour).
- 3. Calculating the depth for each type of scour.
- 4. Accounting for the cumulative effects of each type of scour (If more than one type of scour is present, the effects of the scour types are additive.)
- 5. Reviewing the calculated scour depth for accuracy based on: experience from similar streams; conditions noted during the field visit; and an understanding of the calculations.

3.3 Types of Scour

Five types of scour are defined below¹¹: Bend Scour, Local Scour, Constriction Scour, Drop/Weir Scour, and Jet Scour.

Local Scour – Local scour appears as discrete and tight scallops along the bank line, or as depressions in the streambed. It is generated by flow patterns that from around an obstruction in a stream and spill off to either side of the obstruction, forming a horseshoe-shaped scour patter in the streambed. When flow in the stream encounters an obstruction, for example a bridge pier; the flow direction changes. Instead of moving downstream, it dives in front of the pier and creates a roller (a secondary flow pattern) that spills off to either side of the obstruction. The resulting flow acceleration and vortices around the base of the obstruction result in higher erosive forces around the pier, which move more bed sediment, thereby creating a scour hole¹². The location around the pier is scoured because the bed is eroded deeper at the pier than the bed of the stream adjacent to it.

Bend Scour – When flow moves along a bend, the thalweg (the deepest part of the streambed) shifts to the outer corner of the channel and pronounced bend scour occurs near the outer edge of the channel. Bend scour results from accelerating and spiraling flow patterns found in the meander bend of a stream. Sharper meander bends generate deeper scour than gentle bends. The maximum shear stress acting on a bend can be two or more times as high as the shear stress acting on the bed⁷.

Constriction Scour – Constriction scour occurs when features along the streambank create a narrower channel than would normally form. Often the constricting feature is "harder" than the upstream or downstream bank and can resist the higher erosive forces generated by the constriction. Bedrock outcrops often form natural constrictions. The average velocity across the

width of the channel increases, resulting in erosion across the entire bed of the channel at the constriction. If the bed material is erodible, the channel bed at the constricted section may be scoured deeper than the channel bed upstream or downstream. Large wood jams or bridge abutments are common examples of features that cause constriction scour. Bank features such as rocky points or canyon walls, overly narrow, man-made channel widths (e.g., with groins), or well-established tree roots on a streambank in smaller channels can cause constriction scour.

Drop/weir Scour – Drop/weir scour is the result of plunging vertical flow as water pours over a raised ledge or a drop into a pool, crating a secondary flow pattern known as a roller. The roller scours out the bed below the drop. Energy-dissipation pools may result from drop scour. Pools below perched culverts, spillways, or natural drops (such as those found in high gradient mountain streams), are all causes of drop scour.

Jet Scour – Jet scour occurs when flow enters the stream in the same manner as flow ejecting from the nozzle of a hose. The entering flow could be submerged, or could impact the water surface from above. The impact force from the flow results in jet scour on the streambed and/or bank. Lateral bars, subchannels in a braided or side channel or tributary, or an abrupt channel bend can also create jet scour.

Because scour equations are type-specific, the first step in determining the potential depth of scour is to identify the types of scour that occur at the project site. For instance, an equation for calculating *Local Scour* will give an incorrect depth if applied to a site affected only by *Constriction Scour*. A combination of multiple scour mechanisms could be occurring and all must be identified and accounted for.

All of the scour equations presented are empirical. Empirical equations are based on repetitious experiments or measurements in the field, and therefore, can be biased towards a specific type of stream from which the measurements were made. In general, however, empirical equations are developed with the intention to error on the conservative side if applied correctly.

The scour equations may distinguish between *live-bed* and *clear-water conditions*. These categories refer to the sediment loading during the design event. *Live-bed conditions* exist when stream flow is transporting sediment at or near its capacity to do so. Under such conditions, erosion is somewhat offset by deposition, as stream flow needs to "drop" sediment in order to "pick up" new sediment. *Clear-water conditions* exist when stream flow is transporting sediment at a rate that is far below its capacity to do so. Such conditions often occur downstream of dams or sediment detention basins. Because clear-water stream flow is "sediment starved," it has the capacity to entrain and transport sediment without associated deposition. Accordingly, clear-water conditions usually produce deeper scour depths than live-bed conditions.

3.3.1 Local Scour

Research on scour has focused on local scour at bridge piers and abutments. If the geometry of an obstruction, such as a boulder or rootwad, can be equated to the geometry of a pier, then pier scour equations are applicable. If the location and shape of the obstruction more closely

resembles a bridge abutment rather than a pier, then scour equations for bridge abutments should be used. Obstructions that resemble bridge abutments include large wood installations, or similar structures, that are attached directly to the streambank. Equations for estimating pier and abutment scour are presented below.

3.3.2 Estimating Pier Scour

Numerous equations are available for predicting scour depths near piers. In general, these equations have been developed for sand-bed rivers. However, when applied to streams with larger size bed material (i.e., gravel-bed streams), these equations will tend to give conservative results. The likelihood of the scour depths predicted by these equations being actualized is probabilistic. Predicted depth of scour may not be entirely achieved, may take quite a long time to occur, or may occur during the first large flood.

The pier scour equation presented below includes an adjustment for bed materials that have a D_{50} of 6 cm or larger, and thus is applicable to gravel-bed streams. Judgment should be used to adjust the calculated value as appropriate based on observed stream conditions. In addition, the results of Equation (24) can be used to double-check the results of the pier scour analysis.

When using a pier scour equation to estimate scour near an obstruction, the obstruction must be represented as a pier. For instance, a boulder may be represented in the equation by a cylindrical pier of equal diameter. A log or rootwad may be represented as a round or square-nosed pier of the appropriate length. Note that the pier scour equations assume that the pier extends upwards beyond the water surface. When pier scour depth is calculated for obstructions that do not extend to the water surface (under the analyzed flow), the resulting scour depth should be reduced slightly, according to the judgment of the engineer.

One of the more commonly applied and referenced pier scour equations is the CSU (Colorado State University) equation presented below¹³. The CSU Equation does not differentiate between live-bed and clear-water scour, and is recommended for the analysis of both conditions. In addition, the CSU Equation includes a correction factor (K₄) to adjust for bed materials of D_{50} greater than or equal to 6 cm.

CSU Equation for piers

$d / y_1 = 2.0 K_1$	K ₂ K ₃ K ₄ (b/y ₁	$^{0.65}$ Fr ^{0.43}	(Equation 12)
where:	$y_1 = $ flow dept b = pier width	depth of scour below local streambed elevate h directly upstream of the pier (m) (m) (Hydraulics Figure 5) umber: $V / (g y)^{0.5}$ (dimensionless)	ion (m)
	Where:	V = velocity of flow approaching the abutm g = acceleration due to gravity (9.81 m/s ²) y = flow depth at pier (m)	ent (m/s)

K₁ through K₄ are as defined below

Note that for the special case of round-nosed piers aligned with the flow:

 $\label{eq:constraint} \begin{array}{l} d \leq 2.4 \text{ times the pier width for } Fr \leq 0.8 \\ d \leq 3.0 \text{ times the pier width for } Fr > 0.8 \end{array}$

 K_1 = Correction factor for pier nose shape:

For approach flow angle > 5 degrees, $K_1 = 1.0$ (**Hydraulics Figure 5**) For approach flow angle \leq 5 degrees:

square nose	$K_1 = 1.1$
round nose	$K_1 = 1.0$
circular cylinder	$K_1 = 1.0$
group of cylinders	$K_1 = 1.0$
sharp nose	$K_1 = 0.9$

 K_2 = Correction factor for approach flow angle

 $K_2 = (\cos \theta + L/b \sin \theta)^{0.65}$

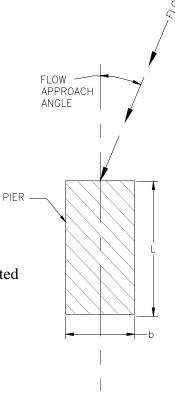
where: $K_2 = \text{correction factor from Hydraulics Table 3}$ L = length of the pier which is being directly subjectedto impinging flow at the approach angle (m) (Hydraulics Figure 5) $\theta = \text{flow angle of approach to pier (in degrees)}$ Maximum L/b = 12

Hydraulics Table 3. K₂ vs. L/b

θ	L/b = 4	L/b = 8	L/b = 12
0	1	1	1
15	1.5	2	2.5
30	2	2.8	3.5
45	2.3	3.3	4.3
90	2.5	3.9	5

Hydraulics Figure 5. Pier scour flow approach angle.

 K_3 = Correction factor for bed conditions, based on dune height, where dunes are repeating hills formed from moving sand across the channel bed. See **Hydraulics Table 4**.



Bed Conditions	Dune height (m)	K ₃
clear water scour	N/A	1.1
plane bed & antidune flow	N/A	1.1
small dunes	0.6 to 3	1.1
medium dunes	3 to 9	1.1 to 1.2
large dunes	9≥	1.3

Hydraulics Table 4. K₃ based on bed conditions

For gravel-bed rivers, the recommended value of K_3 is 1.1

 K_4 = Correction factor for armoring of bed material (scour decreases with armoring)

 K_4 range = 0.7 to 1.0

 $K_4 = 1.0, \ \text{for} \ D_{50} \ < 0.06 \ \text{m, or for} \ V_r > 1.0$

$$K_4 = [\ 1 \ \text{-} \ 0.89 \ (1 \ \text{-} \ V_r \)^2 \]^{0.5}, \ \text{for} \ D_{50} \ge 0.06 \ m,$$

where:

$$\begin{split} V_r &= (V - V_i) / (V_{c90} - V_i) \\ V_i &= 0.645 \ (D_{50} / b)^{0.053} \ V_{c50} \\ V_c &= 6.19 \ y_1^{1/6} \ D_c^{1/3} \end{split}$$

and:

$$\begin{split} V &= approach \ flow \ velocity \ (m/s) \\ V_r &= velocity \ ratio \\ V_i &= approach \ velocity \ when \ particles \ at \ a \ pier \ begin \ to \ move \ (m/s) \\ V_{c90} &= critical \ velocity \ for \ D90 \ bed \ material \ size \ (m/s) \\ V_{c50} &= critical \ velocity \ for \ D50 \ bed \ material \ size \ (m/s) \\ g &= acceleration \ due \ to \ gravity \ (9.81 \ m/s^2) \\ D_c &= critical \ particle \ size \ for \ the \ critical \ velocity, \ V_c \ (m) \\ y_1 &= \ flow \ depth \ directly \ upstream \ of \ the \ pier \ (m) \end{split}$$

3.3.2.1 Top Width of Scour Hole at Pier

USDOT¹³ recommends using 2 times the scour depth as a reasonable estimate of scour hole top width in cohesionless materials such as sands and gravels. Scour hole top width is measured from the edge of the pier to the outside edge of the adjacent scour hole.

3.3.2.2 Estimating Scour at Abutments

Like pier scour equations, abutment scour equations have generally been developed for sand-bed rivers. When applied to streams with larger size bed material (i.e., gravel-bed streams), these equations will tend to give conservative results. The scour depths predicted by these equations

2004 Stream Habitat Restoration Guidelines: Final Draft

may not occur, or may take quite a long time to occur, on gravel-bed streams. As USDOT¹⁴ reports: "reliable knowledge of how to predict the decrease in scour hole depth when there are large particles in the bed material is lacking." Nonetheless, the equations that are available work for sand-bed rivers, and their results, yield a conservative estimate for scour depth on gravel-bed streams. As always, judgment should be used to adjust the calculated value as needed based on observed stream conditions. On coarse-grained streams, this will usually mean reducing the calculated value somewhat. The results of Equation (24) can be used to double-check the results of the abutment scour analysis.

The Froehlich Equation¹³ presented below can be used to estimate scour at an abutment or abutment-like structure. Several variables are included in the equation to describe parameters such as the abutment shape, angle with respect to flow, and abutment length normal to the flow direction. When using this equation to calculate scour for a structure such as a logjam, these parameters should be used, along with good judgment, to describe the structure as best as possible. Note that the abutment scour equation assumes that the abutment extends upwards beyond the water surface. When abutment scour depth is calculated for obstructions that do not extend to the water surface (under the analyzed flow), the resulting scour depth should be reduced slightly, according to the judgment of the engineer.

Froehlich Equation for Live Bed Scour at Abutments

$$d / y = 2.27 K_1 K_2 (L' / y)^{0.43} Fr^{0.61} + 1.0$$

where:

 $\begin{array}{l} d = \mbox{maximum depth of scour below local streambed elevation (m)} \\ y = \mbox{flow depth at abutment (m)} \\ K_1 = \mbox{ Correction factor for abutment shape} \\ & \mbox{vertical abutment = 1.0} \\ & \mbox{vertical abutment with wing walls = 0.82} \\ & \mbox{spill through abutment = 0.55} \\ K_2 = \mbox{ Correction factor for angle of embankment to flow = (θ / 90)$^{0.13} \\ & \mbox{where } \theta = \mbox{angle between the downstream channel bank line and alignment of the abutment} \\ & \mbox{$\theta > 90$ degrees if embankment points upstream} \\ & \mbox{$\theta < 90$ degrees if embankment points downstream} \\ & \mbox{ L' = length of abutment projected normal to flow (m)} \\ & \mbox{ L' = A / y} \end{array}$

Where: A =flow area of approach cross section obstructed by the embankment (m²)

(Equation 13)

Fr = Froude number of flow upstream of abutment

1.0 is added as a safety factor.

3.3.2.3 Clear-Water Scour at an Abutment

USDOT recommends using the live-bed equation presented above to calculate clear-water scour.

3.3.3 Bend Scour

Scour occurs on the outside of channel bends due to spiraling flow as described previously. Bend scour removes materials from the bank toe, potentially precipitating general bank erosion or mass failure.

Quick Methods

Bend scour can be quickly estimated using the following two methods. Field observation/measurement of scour at established bends can yield a quick indication of the magnitude of scour to be expected if correlated to the flows that produced the scour. A first estimate can also be obtained by assuming the scour in any given bend to be about equal to the flow depth found immediately upstream and downstream of the bend¹⁵. This estimate will be somewhat conservative for mild bends.

3.3.3.1 Calculation Methods

Research on scour in bends has produced several empirical equations. Below are three such methods by Thorne, Maynord and Wattanabe. When used with professional engineering judgment, these equations should produce reasonable estimates of bend scour. Please pay particular attention to the notes related to each method and select a method for design based on the appropriateness for the given conditions.

Thorne Equation

Hoffmans and Verheij¹⁵ presented the following equation developed by Thorne based on flume and large river experiments. The mean bed particle size varied from 0.3 to 63 mm. This equation is applicable to gravel-bed streams. Metric or English units may be used.

$$d/y_1 = 1.07 - \log(R_c/W - 2)$$
 for $2 < R_c/W < 22$ (Equation 14)

where: d = maximum depth of scour below local streambed elevation (m or ft)

 y_1 = average flow depth directly upstream of the bend (m or ft) W = width of flow

 R_c = channel radius of curvature at channel centerline (m or ft)

The width of flow in Equation (14) corresponds to the width of active flow. This width is subject to engineering judgment, however, this width often corresponds to the bankfull top width for streams that are flowing near or above bankfull stage.

Maynord Equation

Maynord¹⁶ reviewed bend scour estimates for natural, sand-bed channels and presented one bend scour equation by Wattanabe and a second method by S. Maynord. The Maynord and Wattanabe equations are listed below. These equations are useful for predicting scour depths on sand-bed streams and for determining conservative scour depths (for comparison to other methods) on streams with coarser bed materials.

 $D_{mb}/D_u = 1.8 - 0.051 (R_c/W) + 0.0084 (W/D_u)$

(Equation 15)

where: D_{mb} = maximum water depth in bend D_u = mean channel depth at upstream crossing (area/W) R_c = centerline radius of bend W = width of flow at upstream end of bend

Notes:

- Equation 15 was developed from measured data on 215 sand-bed channels.
- The data were biased for flow events of 1-5 yr return intervals.
- Equation will not apply when higher return intervals occur that cause overbank flow exceeding 20% of channel depth.
- There is no safety factor incorporated into this equation- this is the mean scour depth based on the sites measured.
- A safety factor of 1.08 is recommended by Maynord.
- The equation is limited to: $1.5 < R_c/W < 10$ (use $R_c/W = 1.5$ when < 1.5), and limited to: $20 < W/D_u < 125$ (use $W/D_u = 20$ when < 20). English or metric units may be used
- The width of flow in Equation 15 corresponds to the width of active flow. This width is subject to engineering judgment. However, this width often corresponds to the bankfull top width for streams that are flowing near or above bankfull stage.

Wattanabe Equation

$$\begin{split} d_s / D &= \alpha + \beta \ (W/R_c) \end{split} \tag{Equation 16} \\ \text{Where: } \alpha &= 0.361 \ X^2 - 0.0224 X - 0.0394 \\ X &= \log_{10} \ (WS^{0.2}/D) \\ S &= \text{ bed slope;} \\ d_s &= \text{ scour depth below maximum depth in unprotected bank;} \\ W &= \text{ channel top width (water surface width)} \\ D &= \text{ mean channel depth (area/W);} \\ \beta &= 2/(\pi \ 1.226 \ ((1/\sqrt{f} \) - 1.584) \ x) \\ f &= \text{ Darcy friction factor} \\ x &= 1/ \ [\ 1.5 \ f \ \{(1.11/\sqrt{f} \) - 1.42\} \ \sin \sigma + \cos \sigma \] \\ \sigma &= \tan^{-1} \ [\ 1.5 \ f \ ((1.11/\sqrt{f} \) - 1.42)] = \text{ Darcy friction factor} = 64/Re \\ & \text{where } \text{Re} = \text{Reynolds number} \end{split}$$

Notes:

- Results correlate well with Mississippi River data and under predicted Thorne and Abt data (1993) by about 25%.
- Limits of application are unknown.
- A safety factor of 1.2 is recommended with this method.
- English or metric units may be used

3.3.4 Constriction Scour

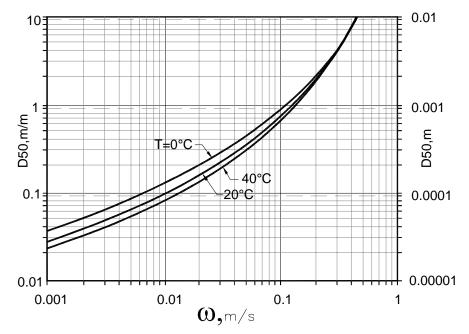
Constriction Scour equations were primarily developed from flume tests with the constriction resulting from bridge abutments. However, these equations apply equally well to natural constrictions or constrictions caused by installation of instream structures such as groins.

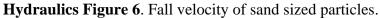
3.3.4.1 Live-bed constriction scour

The following equation for live-bed constriction scour was developed primarily for sand-bed streams. Its application to gravel-bed streams is useful in two ways:

- 1. It provides a conservative estimate of scour depth, and
- 2. It can, by extrapolation of the data in **Hydraulics Figure 6**, provide scour depth estimates for streams with gravel-sized bed materials.

Coarse sediments in the bed may limit live-bed scour. When coarse sediments are present, it is recommended that scour depths, under live-bed and clear-water conditions, (see next section) be calculated and that the smaller of the two calculated scour depths be used. As always, judgment should be used to adjust the calculated value as appropriate based on experience and observed stream conditions. On coarse-grained streams, this will usually mean reducing the calculated value somewhat.





Laursen Equation for Live-Bed Conditions¹³

$$y_2 / y_1 = (Q_2/Q_1)^{0.86} (W_1/W_2)^A, \qquad d = y_2 - y_0$$

(Equation 17)

where:

d = average depth of constriction scour (m)

 y_0 = average depth of flow in constricted reach without scour (m)

 y_1 = average depth of flow in upstream main channel (m)

 y_2 = average depth of flow in constricted reach after scour (m)

 $Q_2 =$ flow in constricted channel section (m³/s)

 $Q_1 =$ flow (m³/s) in upstream main channel (disregard floodplain flow)

 W_1 = channel bottom width at upstream cross section (m)

 W_2 = channel bottom width in constricted reach (m)

A = exponent from Hydraulics Table 5

Hydraulics Table 5. Exponent "A" based on U_*/ω

U*/w	Α	Mode of Bed Material Transport
< 0.5	0.59	Mostly bed load
0.5 to 2.0	0.64	Mostly suspended load
> 2.0	0.69	Mostly suspended load

 ω = fall velocity (m/s) of bed material based on D₅₀ (see Hydraulics Figure 5) U* = shear velocity = (g y₁ S_e)^{0.5} (m/s)

> where: g = acceleration due to gravity (9.81 m/s²) S_e = slope of energy grade line in main channel

Notes:

- 1. As presented here, this equation assumes that all stream flow passes through the constricted reach.
- 2. In review, coarse sediments in the bed may limit live-bed scour. When coarse sediments are present, it is recommended that scour depths under live-bed and clear-water conditions (see following equation) both be calculated and that the smaller of the two calculated scour depths be used.

3.3.5 Clear-water conditions

The following equation calculates constriction scour under clear-water conditions. Unlike the live-bed equation presented above, this equation makes allowance for coarse bed materials.

Laursen Equation for Clear-Water Conditions¹³

$$y_2 = \{ 0.025 Q_2^2 / [D_m^{0.67} W_2^2] \}^{0.43}, \quad d = y_2 - y_0$$
 (Equation 18)

where: d = average depth of constriction scour (m)

 y_0 = average depth of flow in constricted reach without scour (m)

 y_2 = average depth of flow in constricted reach after scour (m)

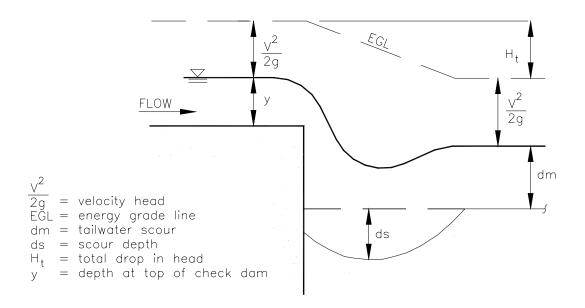
 $Q_2 =$ flow in constricted channel section (m³/s)

 $D_m = 1.25D_{50} =$ assumed diameter of smallest non-transportable particle in the bed material in the constricted reach (m)

 W_2 = channel bottom width in constricted reach (m)

3.3.6 Drop/Weir Scour

Two equations are presented here for estimating scour depths for a flow pouring over a weir, step pool, grade control structure, or drop structure. **Hydraulics Figure 7** shows the typical configuration of such structures. The equations were developed to estimate scour immediately downstream of vertical drop structures and sloping sills.



Hydraulics Figure 7. Schematic of a vertical drop caused by a drop structure.

True vertical drop structures typically include weirs and check dams constructed of materials able to maintain sharp, well-defined crests over which streamflow spills. Check dams and weirs constructed of logs and tightly constructed rock can create hydraulic conditions associated with vertical drop structures. Structures constructed of loose rock usually form a sloping sill.

Equation (19) is recommended for predicting scour depth immediately downstream of a vertical drop structure and for determining a conservative estimate of scour depth for sloping sills. Equation (20) specifically addressed sloping sills constructed of rock. When designing check dams, weirs, grade controls, and similar structures, it is recommended that the designer utilize these equations as applicable (using professional judgment) to estimate expected scour depth immediately downstream of the structure.

U.S. Bureau of Reclamation Equation – Vertical Drop Structure¹⁰

$$d_s = K H_t^{\ 0.225} q^{0.54} \text{ - } d_m$$

(Equation 19)

where:

e: d_s = local scour depth (below unscoured bed level) immediately downstream of vertical drop (m)

- q = discharge per unit width (m³/s/m)
- H_t = total drop in head, measured from the upstream to downstream energy grade line (m)

 d_m = tailwater depth immediately downstream of scour hole (m) K = 1.9

The depth of scour calculated in Equation (19) is independent of bed material grain size. If the bed contains large or resistant materials, it may take years or decades for scour to reach the depth

calculated in Equation (19). Alternatively, less durable bed materials and/or large flow events may lead to very rapid scour.

Laursen and Flick Equation – Sloping Sill¹⁷

$$d_{s} = \{ [4 (y_{c}/D_{50})^{0.2} - 3 (R_{50}/y_{c})^{0.1}] y_{c} \} - d_{m}$$
 (Equation 20)

where:

$d_s = local scour depth (below unscoured bed level) immediately downstream of$
vertical drop (m or ft)
$y_c = critical depth of flow (m or ft)$
D_{50} = median grain size of material being scoured (m or ft)
R_{50} = median grain size of stone that makes up the grade control, weir, or check
dam (m or ft)
d_m = tailwater depth immediately downstream of scour hole (m or ft)

Equation (20) predicts scour depth at the base of a sloping sill with slope of 1V:4H. This equation can be used to estimate scour at the base of a short riffle, or similar ramp-like structure.

3.3.7 Jet Scour

Although jet scour is a phenomenon associated with streams, it is not typically a component of streambank or instream structure design. In special cases where jet scour may be desirable or unavoidable, analysis is necessary, so the designer should consult a hydraulic design manual such as Simons & Senturck¹⁸ for guidance. Please refer to the Recommend Reading section of this appendix.

3.3.8 Check Method - Bureau of Reclamation Method

A method developed by the Bureau of Reclamation provides a multi-purpose approach for estimating depths of scour due to bends, piers, grade control structures, and vertical rock banks or walls. The method is usually not as conservative and possibly not as accurate as the individual methods presented above

3.3.8.1 Regime Equations Supported by Field Measurements Method

The Bureau of Reclamation method computes an "average" scour depth by applying a systematic adjustment (Step 2) to the results of three regime equations: the Neil Equation, a modified Lacey equation, and the Blench equation (Step 1).

<u>STEP 1</u>

Neil Equation

Obtain field measurements on an incised reach (one which does not flow overbank except at very high discharge) of the river from which bankfull discharge and hydraulics can be calculated.

Note: Units are metric or English

$$y_{s} = y_{bi} \left(q_{di} / q_{bi} \right)^{m}$$

where:

 y_s = scoured depth below design flow level in incised reach, which is adjusted in Step 2 to yield predicted scour depths (ft or m);

y_{bi} = average bankfull flow depth in incised reach (ft or m);

 q_{di} = design flow discharge per unit width in incised reach (cfs/ft or m³/s/m);

 q_{bi} = bankfull flow discharge per unit width in incised reach(cfs/ft or m³/s/m);

m = exponent varying from 0.67 for sand to 0.85 for coarse gravel.

Modified Lacey Equation

The Lacey equation was modified with the Blench method of zero bed-sediment transport. An incised reach is not required for this application.

Note: Units are metric or English

$$y_L = 0.47 (Q/f)^{0.33}$$
 (Equation 2)

where:

 y_L = mean depth at design discharge (ft or m); Q = design discharge (cfs or m³/s); $f = Lacey's silt factor = 1.76 D_{50}^{0.5}$ where D_{50} is in millimeters

 D_{50} = mean grain size of bed material (must be in mm)

Blench Equation

For zero bed factor (clear water scour)

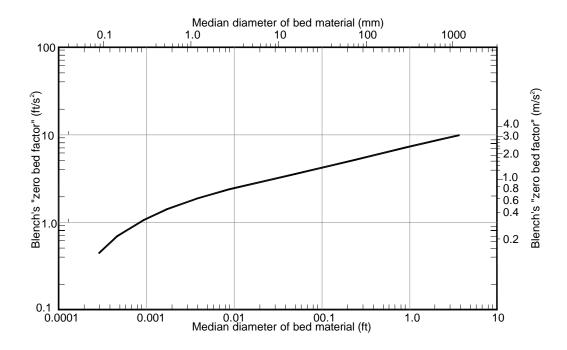
Note: Units are metric or English

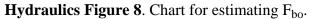
$y_B = q_d^{0.67} / F_{bo}^{0.33}$		(Equation 23)	
where:	y_B = depth for zero bed sediment transport (ft or m) q_d = design flow discharge per unit width (cfs or m ³ /m)		

 F_{bo} = Blench's zero bed factor, from **Hydraulics Figure 8** (ft/s² or m/s²)

(Equation 21)

22)





<u>STEP 2</u> Adjustments to Neil, Modified Lacey, and Blench Results

$$\begin{split} d_{\mathrm{N}} &= K_{\mathrm{N}} \; y_{\mathrm{N}} \\ d_{\mathrm{L}} &= K_{\mathrm{L}} \; y_{\mathrm{L}} \\ d_{\mathrm{B}} &= K_{\mathrm{B}} \; y_{\mathrm{B}} \end{split}$$

where:	d_N, d_L, d_B	= depth of scour from Neil, Modified Lacey, and Blench
		equations respectively;
	K_N, K_L, K_B	= adjustment coefficients for Neil, Modified Lacey, and
		Blench equations as shown in Hydraulics Table 6 .

Hydraulics Table 6. Adjustment coefficients based on channel conditions

Condition	<u>Neill - K_N</u>	Lacey - K _L	<u>Blench - K_B</u>
Bend Scour			
Straight reach (wandering thalweg)	0.5	0.25	0.6
Moderate bend	0.6	0.5	0.6
Severe bend	0.7	0.75	0.6
Right angle bend		1.0	
Vertical rock bank or wall		1.25	
Nose of piers	1.0		0.5 to 1.0
Small dam or grade control across river	0.4 to 0.7	1.5	0.75 to 1.25

(Equation 24)

3.4 Additional Reading

The following texts are recommended reading for those interested in learning the fundamentals of hydraulic analysis.

Chaudhry, M. H. 1993. Open-channel flow. Prentice Hall, Inc., New Jersey. 483 p.

Chow, V. T. 1959. Open-channel hydraulics. McGraw-Hill Inc., New York. 680 p.

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³ Barnes, H. H., Jr. 1967. Roughness Characteristics of Natural Channels. U.S. Geological Survey Water-Supply Paper 1849.

⁴ Hicks, D. M. and P. D. Mason. 1991. Roughness Characteristics of New Zealand Rivers. 336 pp.

⁵ Fischenich, C. 2001. Stability Thresholds for Stream Restoration Materials. EMRRP Technical Notes Collection (ERDC TNEMRRP-SR-29), U.S. Army Engineer Research and Development Center, Vicksburg, MS.

⁶ United States Department of Transportion (USDOT). 1988. Design of Roadside Channels with Flexible Linings. Hydraulic Engineering Circular no. 15. Federal Highway Administration – USDOT.

⁷ Power, M. E., W. E. Deitrich and J. C. Finaly. 1996. Dams and downstream aquatic biodiversity: potential food web consequences of hydrologic and geomorphic change. Environmental Management. 20:887-895.

⁸ Vada, R. L. Jr. and D. L. Weigmann. 1993. The concept of instream flow and its relevance to drought management in the James River basin. Virginia Water Resources Research Center Bulletin. 178.78 pp.

⁹ Cramer, M., K. Bates, D. Miller, K. Boyd, L. Fotherby, P. Skidmore, and T. Hoitsma. 2003. *Integrated Streambank Protection Guidelines*. Co-published by the Washington departments of Fish & Wildlife, Ecology, and Transportation. Olympia, WA. 435pp. Ch 2, Pg 2-9.

¹⁰ Pemberton, Ernest L. and Joseph M. Lara, 1984. Computing Degradation and Local Scour, Technical Guideline for Bureau of Reclamation, Bureau of Reclamation Sedimentation and River Hydraulics Section, Denver, CO. Pg 29-30.

¹¹ Cramer, M., K. Bates, D. Miller, K. Boyd, L. Fotherby, P. Skidmore, and T. Hoitsma. 2003. *Integrated Streambank Protection Guidelines*. Co-published by the Washington departments of Fish & Wildlife, Ecology, and Transportation. Olympia, WA. 435 pp. Ch 2, Pg 2-7 to 2-11.

¹² U.S. Department of Transportation, Federal Highway Administration. 1996. Channel Scour at Bridges in the United States. Publication NO. FHWA-RD-95-184.

¹³ USDOT, 1995. Evaluating Scour at Bridges, Hydraulic Engineering Circular No. 18, Federal Highway Administration, FHWA-IP-90-017, 3rd Ed.

¹⁴ USDOT, 1990. Highways in the River Environment. Federal Highway Administration – USDOT.

¹⁵ Hoffmans, G. J., and Verheij, H. J., 1997. Scour Manual, A. A. Balkema Publishers, Brookfield, VT.

¹⁶ Maynord, S. 1996. "Toe Scour Estimation in Stabilized Bendways", Journal of Hydraulic Engineering, ASCE, v122, n8, p460.

¹⁷ Laursen, E. M., and M. W. Flick, 1983. Final Report, Predicting Scour at Bridges: Questions Not Fully Answered – Scour at Sill Structures, Report ATTI-83-6, Arizona Department of Transportation, 1983.

¹⁸ Simons & Senturck, 1992. Sediment Transport Technology, Water Resources Publications, Littleton, CO.