ES322 EXAMPLE QUANTITATIVE PROBLEM SOLVING TECHNIQUES WITH SOLUTIONS

Primer on Solving Quantitative Style Word Problems in Physical Science and Geology

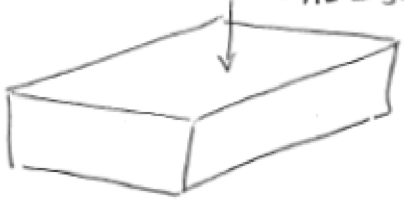
Study Technique: Follow the step-by-step problem solving method outlined below; review the attached example problems and solutions; practice repetitively and compare your answers to key.

(1) Carefully read the word problem
(2) Identify all the variables
    a. ID Knowns
    b. ID Unknowns
(3) Draw and sketch the problem, labeling all the variables with magnitudes and units
(4) Convert all units to consistent dimensions; use unit algebra techniques, cancel units
(5) List all equations that apply to the problem, refer to class notes and text book
(6) Rearrange equations, substitute variables, algebraically solve for the unknown(s)
(7) Check your work, review your math, review your unit algebra, does the value of your answer make logical sense? For example, if you calculate the volume of the Earth as 6 gallons, something is not right, and you need to go back to the drawing board.

Example Problem 1.2 Assume you are dealing with a vertical-walled reservoir having a surface area of 500,000 m^2 and that an inflow of 1.0 m^3/sec occurs. How many hours will it take to raise the reservoir level by 30 cm?

1.2

$A_d = 500{,}000\ m^2$



Inflow = $\left(\frac{1\ m^3}{sec}\right)\left(\frac{60\ sec}{min}\right)\left(\frac{60\ min}{hr}\right) =$

$Q_{inflow} = 3600\ \frac{m^3}{hr}$ = inflow rate

$Vol = A \cdot d$

$d = 30\ cm\ \frac{1\ m}{100\ cm} = 0.3\ m$

$Vol = (500{,}000\ m^2)(0.3\ m) = 150{,}000\ m^3$

$t\ (Q_{inflow}) = \frac{Vol}{\cancel{t}}\ \cancel{t}$

$t\ Q = Vol$

$t = \frac{Vol}{Q} = \frac{150{,}000\ m^3}{3600\ m^3/hr} = \boxed{41.7\ hr}$

UNITS: SI units (Systéme Internationale d'Unités)

Base SI units

length	meter	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

examples of derived units

velocity	m/s
acceleration	m/s^2
area	m^2
density	kg/m^3

Dimensional Analysis $\cdots$ Do the units work out ?

In math class we see quadratic equations of the form

$$y = ax^2 + bx + c$$

all the time and never question its validity. In physics, every quantity has units associated with it (the exception being the ratio of two expressions having the same units). Dimensional analysis allows us to check if the mathematical expression we are using in dimensionally correct. In a short while you'll see the following equation:

$$x(t) = \frac{1}{2}at^2 + v_0 t + x_0$$

where $x(t)$ and x_0 are measured in meters (m), t in seconds (s), the initial velocity v_0 in meters/sec (m/s), and acceleration a in m/s^2. By substituting in the appropriate units for each quantity we observe that

$$m = (\frac{m}{s^2}) * (s)^2 + (\frac{m}{s}) * s + m = m + m + m,$$

verifying that the expression is indeed dimensionally correct.

Had we mistakenly put the squared on the second occurrence of time (t) in the equation rather than the first we'd have

$$x(t) = \frac{1}{2}at + v_0 t^2 + x_0,$$

which would lead to the following dimensionalities:

$$m = (\frac{m}{s^2}) * (s) + (\frac{m}{s}) * (s)^2 + m = \frac{m}{s} + m * s + m.$$

Note that the addition operation is now ill-defined; you can only add quantities of the same type. Thus, checking equations using dimensional analysis helps to catch algebraic mistakes you might make during mathematical manipulations.

Another equation we'll derive when we begin studying projectile motion is known as the range equation:

$$R = \frac{v_0^2 \sin{(2\theta_0)}}{g},$$

where R is the horizontal distance traveled (the Range), v_0 is the initial velocity, θ_0 is the angle it is launched at relative to the horizontal, and g is the acceleration due to gravity. Inserting the units demonstrates that

$$m = \frac{(\frac{m}{s})^2 * 1}{(\frac{m}{s^2})} = m.$$

Note that in our units check I've inserted "1" for the units of the sine function, trigonometric functions are pure numbers, they have no units associated with them.

Powers of ten

Usually we'll use the power of ten that is most convenient for the problem at hand. For a problem involving the thickness of a cell wall, length measurements will often be in terms of nanometers (1 nm $= 10^{-9}$ m), while for a car traveling down the highway a more appropriate unit of measurement might be the kilometer (1 km $= 10^{3}$ m). Some commonly used unit prefixes and powers of ten:

pico	p	10^{-12}	tera	T	10^{12}
nano	n	10^{-9}	giga	G	10^{9}
micro	μ	10^{-6}	mega	M	10^{6}
milli	m	10^{-3}	kilo	k	10^{3}
centi	c	10^{-2}			

Unit Conversions $\cdots$ always multiply by one!

Basic idea: since 1 minute = 60 seconds, divide each side by 60 seconds to get:

$$\frac{1 \text{ minute}}{60 \text{ seconds}} = \frac{60 \text{ seconds}}{60 \text{ seconds}} = 1$$

We could just have easily divided by 1 minute to arrive at:

$$\frac{60 \text{ seconds}}{1 \text{ minute}} = \frac{1 \text{ minute}}{1 \text{ minute}} = 1.$$

Since multiplying by 1 leaves the value of a quantity unchanged, we can accomplish our conversions from one system of units to another by multiplying by the "appropriate form" of 1.

Example: lets convert 55 miles/hour to meters/second.

$$\frac{55\text{ mi}}{1\text{ hr}}=\left(\frac{55\ \cancel{\text{mi}}}{1\ \cancel{\text{hr}}}\right)\left(\frac{1\ \cancel{\text{hr}}}{60\ \cancel{\text{min}}}\right)\left(\frac{1\ \cancel{\text{min}}}{60\text{ sec}}\right)\left(\frac{5280\ \cancel{\text{ft}}}{1\ \cancel{\text{mi}}}\right)\left(\frac{12\ \cancel{\text{in}}}{1\ \cancel{\text{ft}}}\right)\left(\frac{2.54\ \cancel{\text{cm}}}{1\ \cancel{\text{in}}}\right)\left(\frac{1\text{ m}}{100\ \cancel{\text{cm}}}\right)$$

$$\frac{55\text{ mi}}{1\text{ hr}}=\left(\frac{55*5280*12*2.54}{60*60*100}\right)\left(\frac{\text{m}}{\text{s}}\right)$$

$$\text{Thus }\frac{55\text{ mi}}{1\text{ hr}}=24.6\frac{\text{m}}{\text{s}}.$$

Another example: convert 2.7 grams/cm^3 to kg/m^3.

$$\frac{2.7\text{ gr}}{1\text{ cm}^3}=\left(\frac{2.7\ \cancel{\text{gr}}}{1\ \cancel{\text{cm}^3}}\right)\left(\frac{1\text{ kg}}{10^3\ \cancel{\text{gr}}}\right)\left(\frac{10^2\ \cancel{\text{cm}}}{1\text{ m}}\right)^3$$

$$\text{Thus }\frac{2.7\text{ gr}}{1\text{ cm}^3}=\left(\frac{2.7*10^6}{10^3}\right)\left(\frac{\text{kg}}{\text{m}^3}\right)=2.7*10^3\left(\frac{\text{kg}}{\text{m}^3}\right)$$

G476/576 Hydrology
Introduction to Applied Problems in Hydrology

Determine the following, show all of your math work and unit algebra.

1. A city has a reservoir with vertical sides and a surface area of 12.3 acres. Following the rainy season, the reservoir is filled to a depth of 3.0 m. During the dry season, the reservoir loses 3.5 in of water per week (wk) to evaporation. At the same time, the city pumps water from the reservoir at a rate of 100 gal/day. What volume of water will remain in storage after 3 weeks into the dry season? (answer in cubic meters, and gallons)

2. How long must a pump with a capacity of 25 gal/min pump to fill a tank with a capacity of 60 cubic meters?

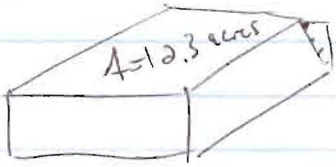
3. A small urban watershed has an area of 20.5 square miles. A summer storm drops an average of 2.5 in of rain over the entire watershed. If 65% of the rainfall runs off the watershed into surface-water bodies, what is the volume of runoff (show all of your work and unit algebra):

A. In cubic inches?

B. In cubic feet?

C. In cubic meters?

Applied Problems in Hydrology

1) 

rainy season: 3.0m deep
dry season: $Q_{evap} = 3.5$ in/wk
$Q_{inflow} = 100$ gal/day

How much H_2O remains after 3 weeks?
- m^3
- gal

$\cancel{12.3 \text{ acres} \times \frac{2.47\times10^{-4} \, m^2}{\text{acre}} \times 3.0\,m = 9.11\times10^{-3} \, m^3}$

a) $12.3 \text{ acres} \times \frac{4047 \, m^2}{\text{acre}} \times 3.0\,m = \boxed{1.49\times10^5 \, m^3} = \text{Vol}$

b) $Q_{evap} =$ ~~3.5 acre-inches~~ $3.5 \text{ in} \times 12.3 \text{ acres} = 43.1$ acre-inches

$43.1 \text{ acre-inches} \times \frac{1 \text{ ft}}{12 \text{ in}} = 3.59$ ac-ft

$3.59 \text{ ac-ft} \times \frac{1233.5 \, m^3}{\text{ac-ft}} = \boxed{4425 \, m^3/\text{wk}}$

c) $Q_{inflow} = 100 \, \text{gal}/\text{day} \times \frac{7 \text{ days}}{\text{week}} \times \frac{3.78\times10^{-3} \, m^3}{\text{gal}} = 2.646 \, m^3/\text{wk}$

d) $Q_{inflow} - Q_{evap} = 2.646 \, m^3/\text{wk} - 4425 \, m^3/\text{wk} = -4422.4 \, m^3/\text{wk}$

$-4422.4 \, m^3/\text{wk} \times 3 \text{ wk} = -13267 = -1.3267\times10^4 \, m^3$

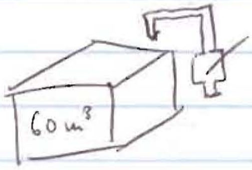
↓

$.13267\times10^5 \, m^3$

e) $1.49\times10^5 \, m^3 - 0.13267\times10^5 \, m^3 = \boxed{1.36\times10^5 \, m^3}$

$1.36\times10^5 \, m^3 \times \frac{264.17 \text{ gal}}{m^3} = 3.59\times10^7$ gal

2. vol $= 60\ m^3$ flux $= 25$ gal/min.



$t_o = 0.0\ m^3$

$t_{final} = 60\ m^3$

a) $25\ \cancel{gal}/min \times \frac{3.78\times10^{-3}\ m^3}{\cancel{gal}} = 9.45\times10^{-2}\ m^3/min$

b) $\frac{Vol}{\cancel{Q}\ Q} = t,\ \frac{60\ \cancel{m^3}}{9.45\times10^{-2}\ \cancel{m^3}/min} = 635\ min \times \frac{1 hr}{60\ min} = \boxed{10.6\ hrs}$

3.

$A = 20.5\ mi^2$

$d = 2.5\ in$, 65% runoff

$vol = \left(20.5\ mi^2 \times \left(\frac{5280\ ft}{mi}\right)^2 \times \left(\frac{12\ in}{ft}\right)^2\right) \times 2.5\ in = 2.06\times10^{11}$

a) $2.06\times10^{11}\ in^3 \times \left(\frac{65}{100}\right) = \boxed{1.34\times10^{11}\ in^3 \text{ runoff}}$

b) $1.339\times10^{11}\ in^3 \times \left(\frac{1\ ft}{12\ in}\right)^3 = \boxed{7.75\times10^{7}\ ft^3 \text{ runoff}}$

c) ~~$1.339\times10^{11}\ in^3 \times \frac{61,023\ in^3}{m^3}$~~

$1.339\times10^{11}\ \cancel{in^3} \times \frac{m^3}{61,023\ \cancel{in^3}} = \boxed{2.19\times10^{6}\ m^3 \text{ runoff}}$

TOPOGRAPHIC MAPS - REVIEW

3. What is the scale (stated as a ratio) of a map where 1inch = 1 mile? Show your calculations.

$1\text{ mi} \cdot \frac{5280\text{ ft}}{1\text{ mi}} \cdot \frac{12\text{ in}}{1\text{ ft}} = 63,360$

1:63,360

4. On a map drawn to a scale of 1inch = 1 mile, what distance on the map represents 2,000 feet? Show your calculations.

$2000\text{ ft} \cdot \frac{12\text{ in}}{1\text{ ft}} = \frac{24,000\text{ in}}{63,360\text{ in}} = 0.38\text{ in}$

5. What is the scale (stated as a ratio) of a map where 1" = 2,000'? Show your calculations?

$2000\text{ ft} \cdot \frac{12\text{ in}}{1\text{ ft}} = 24000$

1:24000

6. On a map drawn to a scale of 1:100,000, what distance is represented by 3 inches? Show your calculations.

$3\text{ in} \cdot 100,000\text{ in} = 300,000\text{ in} \cdot \frac{1\text{ ft}}{12\text{ in}} \cdot \frac{1\text{ mi}}{5280\text{ ft}} = 4.73\text{ mi}$

7. On a map drawn to a scale of 1:100,000, what distance is represented by 3 cm? Show your calculations.

$3\text{ cm} \cdot \frac{1\text{ in}}{2.54\text{ cm}} = 1.18\text{ in} \cdot 100,000\text{ in} = 118,000\text{ in} \cdot \frac{1\text{ ft}}{12\text{ in}} \cdot \frac{1\text{ mi}}{5280\text{ ft}} = 1.86\text{ mi}$

8. A 4"-long ridge on an air photo is 2 miles on a map. What is the photo scale?

$4'' = 2\text{ mi} \cdot \frac{5280\text{ ft}}{1\text{ mi}} \cdot \frac{12\text{ in}}{1\text{ ft}} = \frac{126,720\text{ in}}{4\text{ in}} = 31680$

1:31680

INTRODUCTION TO TOPOGRAPHIC MAPS

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All of the following questions refer to the Monmouth, OR Quadrangle.

1) What is the fractional scale, contour interval, and magnetic declination of this map?
a) Scale: 1:24,000 b) Contour Interval: 10 ft. c) Declination: 19°E

2) What quadrangle maps are located immediately adjacent to the Monmouth Quad.?
a) North: Rickreal b) South: Lewisburg c) East: Sidney d) West: Airlie North

3) What is the quadrangle size series of this map (in long. and lat.)?
7.5 min

4) What is the date of publication of this map?
1970 (photo revised 1986

5) What does the tick with 4956000m N. mean? (lower right of map)
UTM demarcation

6) What is the name of the major fluvial system flowing through this area. Of What larger drainage basin(s) does this river form a part of?
Williamette River, Columbia River Basin

7) What is the approximate elevation of the Natural Sciences Building based on the map representation? 210 ft

8) Given the fractional scale determine the following

1 in = 24,000 in 12 in = 1 ft 5280 ft = 1 mile

5 inches on the map= 10,000 Feet on ground = 1.89 Miles on ground.
10 inches on the map= 6097.6 Meters on ground = 6.098 Kilometers on ground.

3.28 ft = 1 m 1000 m = 1 km

9) A. What is the road distance in miles along Rt. 99 between Helmick State Park and Monmouth city limits? $= 13 \text{ in} \times 24{,}000 = 312{,}000 \text{ in} \left(\frac{ft}{12 in}\right)\left(\frac{mi}{5280 ft}\right) = 4.92$ miles

B. What is the distance in kilometers?
$4.92 \text{ mi} \left(\frac{5280 ft}{1 mi}\right)\left(\frac{m}{3.28 ft}\right)\left(\frac{km}{1000 m}\right) = 7.93$ km

10) A. Determine the average stream gradients (in Ft/Mi) for the following drainages:

A. Willamette River: Gradient: 177 − 153 = 24 ft Length: 105 − 93 mi = 12 mi 24 ft/12 mi = $\boxed{2 \text{ ft/mi}}$
B. Luckiamute River: Gradient: 212 − 157 = 55 ft Length: 13 − 5 mi = 8 mi 55 ft/8 mi = $\boxed{7 \text{ ft/mi}}$

11) A. What is the highest point of elevation represented on this map? 880 ft.

B. What is the lowest point of elevation represented on this map? 150 ft

C. What is the maximum relief. 880 ft − 150 ft = 730 ft.

.5' / .55" (3.3") = 1.15'
.5' / .375" (3.7") = 1.713'

12) A. What is the longitude and latitude location of the road intersection at Buena Vista
44° 46' 10", 123°, 65' 47"

B. What is the longitude and latitude location of Davidson Hill?
44° 45' 54" 123°, 11' 15"

C. What is the straight line distance in miles between these two points?
$5 \text{ in} \times 24{,}000 = 120{,}000 \text{ in} \left(\frac{ft}{12 in}\right)\left(\frac{mi}{5280 ft}\right) = \boxed{1.89 \text{ miles}}$

D. What is the azimuth bearing FROM Davidson Hill TOWARDS Buena Vista?

085°

E. What is the quadrant bearing FROM Buena Vista TOWARDS Davidson Hill?

S85°W

13) A. What is the nature of the topographic slope in the vicinity of the town of Monmouth?

gently sloping

C. What is the local relief between WOU and the Willamette adjacent to Independence?

210 − 150 = 60 ft

D. Is the outline of the topography east of Independence relatively arcuate or irregular in outline?

irregular

E. What processes might have formed the pattern in D above?

possibly landslides or unstable hill slopes

14) Examine the cultural activity immediately north of Monmouth and Independence.

A. Write a brief assessment of the potential for environmental degradation to the surface and groundwater of this area. List three types of water quality degradation (i.e. contamination) problems that may exist in this area.

One source of environmental degradation that is likely is from agricultural runoff in the area, as agriculture is the predominant land use in the area.

A second source of environmental degradation that may occur is from urban runoff from the the urban sections of the map (the Monmouth / Independence areas)

a third source of potential water contamination that may occur is from industrial runoff from anthropogenic industrial activity in the area & urban center ([illegible]).

18. Determine the elevations of the following locations:

A. Wigrich 260 ft.

B. Oak Hill (SC) 476 ft

C. Dicker Reservoir (NE) 450 ft

D. Davidson Bridge (SC) 160 ft

19. Draw a topographic profile along a line connecting Oak Hill (SC) to Vitae Springs. Use a horizontal scale of 1 in = 4000 Ft, and a vertical scale of 1 in = 333.33 ft (see attached profile paper).

A. Determine the minimum slope grade represented on the profile in percent.

1% Williamette River to Burlington Northern

B. Determine the maximum slope grade represented on the profile in percent.

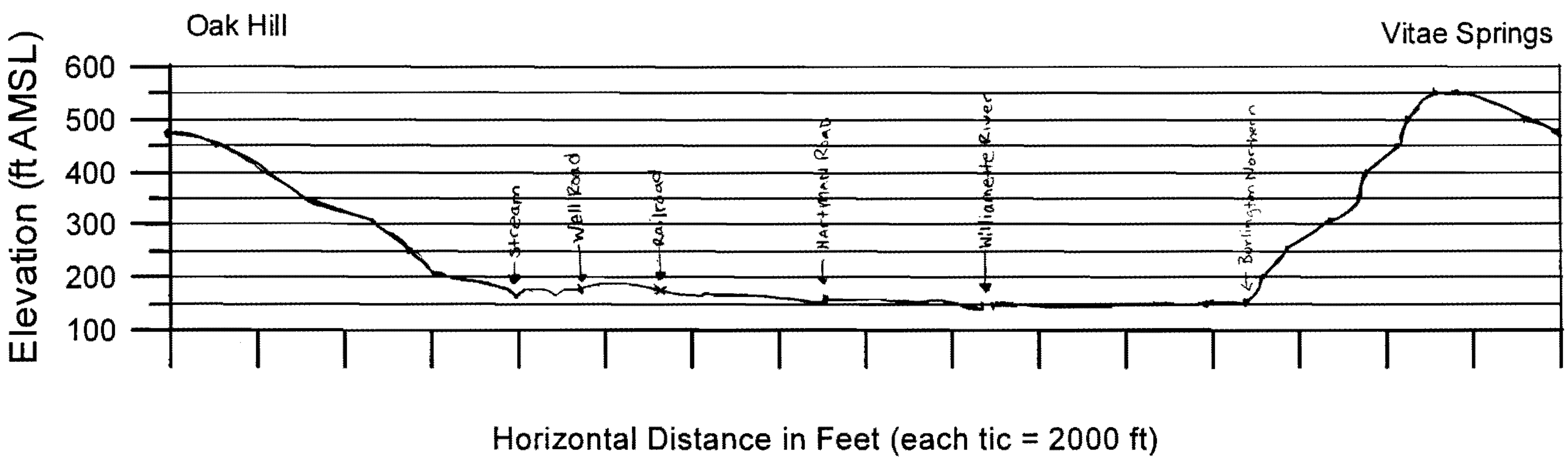
9% Burlington Northern to Vitae Springs

C. Where are the areas most likely associated with flooding?

Williamette River & adjacent

D. The vertical exaggeration of a profile is calculated by: VE = H scale / V scale;
Calculate the vertical exaggeration represented on the attached profile.

Topographic Profile from Oak Hill to Vitae Springs, Monmouth, OR Quad.



Horizontal Scale: 1 in = 4000 ft
Vertical Scale: 1 in = 333.33 ft

V.E. = H/V = $\frac{1/4000}{1/333.33}$ $\boxed{.0833 \text{ ft/ft V.E.}}$

c:wou:geomorph:monblank.grf

G. Time/Evolution/Rates of Change

1. Landscape Evolution: concept of progressive change of landforms in response to surface processes operating over a period of time.

   a. Landforms/landscapes will display characteristic features at successive stages of development.

      (1) Provides and avenue for relative dating of landforms on the basis of developmental stage

         (a) If rates of process/change are known, ages of landforms and landscapes can be determined through deductive reasoning

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In-Class Exercise: Hypothetical Example of a Geomorphic Rate Problem

The High Cascades of Oregon are spectacular mountains comprised of active volcanoes. Volcanic mountains in this area are created by repetitve eruptions of lava and tephra, that build up on the landscape. The average relief between High Cascade volcanic peaks and the older western Cascades is about 7000 ft (in a ball-park kind of way). For the sake of argument, let's assume that the average thickness of an individual eruptive deposit is 20 feet. And that the average High Cascade volcano erupts once every 1500 years. Calculate the following:

(A) What is the average rate of volcanic mountain growth in feet / yr?

$20\ ft/1500\ yr = \boxed{0.013\ ft/yr}$

(B) What is the average rate of volcanic mountain growth in feet / 1000 years?

$0.013 \times 1000 = \boxed{13\ ft}$

(C) What is the average rate of volcanic mountain growth in meters / 1000 years?

$13\ ft \cdot \frac{m}{3.281\ ft} = \boxed{3.96\ m}$

(D) How many years has it taken for the average High Cascade volcano to build its "edifice" above the older western Cascades?

$7000/0.013 = \boxed{538000\ years}$

(E) In that we've only considered volcanic processes building up the Cascade landscape, consider all of the other surface processes that are operating to erode (i.e. lower) the High Cascades. Make a list with a brief discussion.

Rainfall, Wind, and Animals will slowly erode surface bedrock. Gravity can lead to mass wasting, especially when paired with water. Within the last couple hundred years Humans have also had an impact by moving sediment.
