5 Trigonometry

5.1 Introduction

Trigonometry is the study of triangles. Triangles rather than, say, squares or hexagons because any other **polygon** (a closed shape with straight edges) can be constructed by adding triangles together (Fig. 5.1). Thus, if the properties of triangles are understood, any other polygon can also be dealt with.

Triangles are ideal for purposes such as mapping since there are simple rules relating the lengths of their sides to the size of their angles. Figure 5.2 illustrates the quantities which define a given triangle. This triangle has three sides of length a, b and c and three angles of size A, B and C. Note that length a is opposite angle A, b is opposite B and c is opposite C.

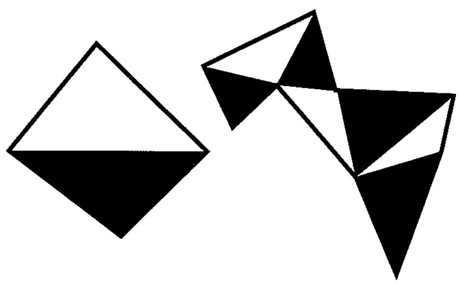


Fig. 5.1 Any polygon can be constructed from a set of triangles.

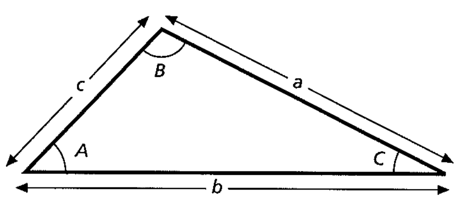


Fig. 5.2 A triangle is described by the length of its three sides and the size of its three angles.

> **Question 5.1** Using a ruler and protractor, sketch the following triangles and determine the unknown three quantities:
> (i) $A = 20°$, $C = 100°$, $a = 4$ cm;
> (ii) $C = 20°$, $a = 3$ cm, $b = 5$ cm.

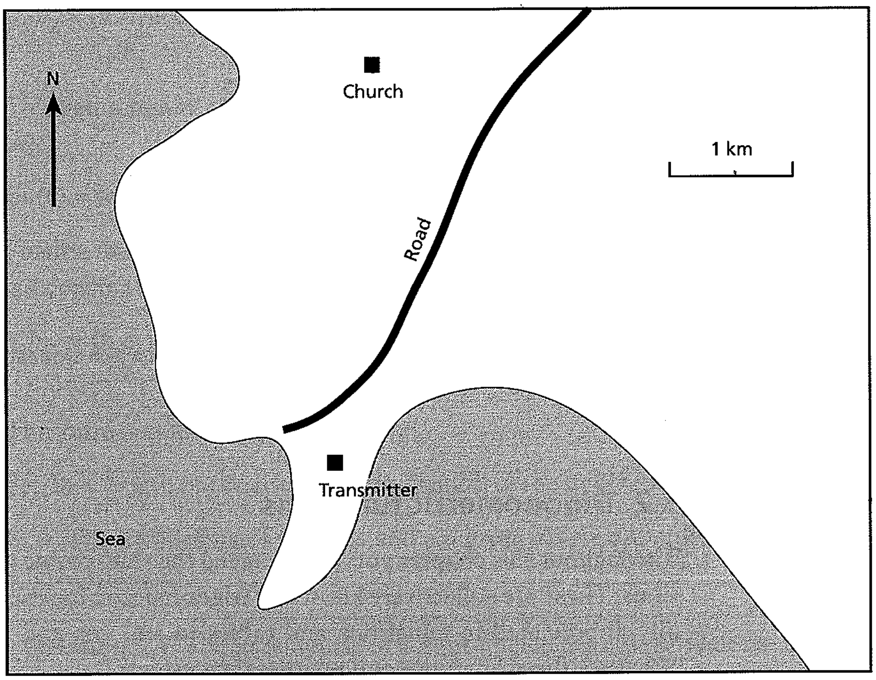


Fig. 5.3 Locate the exposure given the information in Question 5.2.

> **Question 5.2** Examine the map in Fig. 5.3 and measure the distance from the church to the transmitter. If, from an exposure, the church is seen to be located 45° west of north whilst the transmitter is due west, where is the exposure? How far is the exposure from the church and how far from the transmitter?

Angles, in geology, are normally measured in degrees since this is a convenient unit for measuring dips, strikes and other similar quantities. However, there are other units which can be used of which **radians** are the most important. An angle of one radian is about 57.3°. This may seem a very peculiar, and rather large, unit but there are good reasons for its use one of which will be explained in Chapter 8. For now, I will only point out that the radian is defined such that one complete rotation (i.e. an angle of 360°) is 2π radians (~6.28 radians).

> **Question 5.3** Given that 360° is equivalent to 2π radians, what are the following angles in radians? (Hint: What fraction of a complete rotation are these angles?)
> (i) 180°; (ii) 90°; (iii) 270°; (iv) 100°.

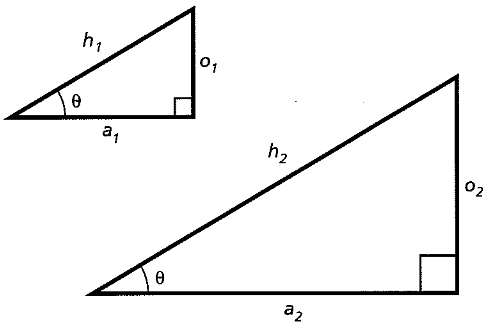


Fig. 5.4 Two similar triangles. The lower triangle has sides which are k times longer than those of the upper triangle. However, all angles are identical.

5.2 Trigonometric functions

Throughout this chapter you will be using the sine, cosine and tangent functions. These are collectively known as **trigonometric functions**. They are usually abbreviated in equations and tables to sin, cos and tan, respectively. What are these functions and why are they useful?

Figure 5.4 shows two **right-angled triangles** (triangles in which one angle is 90°), each of which contains the same angle, θ. However, the second triangle has sides which are k times longer than those of the first, i.e. $o_2 = k.o_1$, $a_2 = k.a_1$ and $h_2 = k.h_1$ where k is a constant. Triangles such as these, which are exactly the same shape but which are of different sizes, are known as **similar** triangles. Incidentally, I have denoted the lengths of the sides using o because this is the side opposite the given angle, a because this is the side adjacent to θ and h for **hypotenuse** which is the side opposite the right angle.

Now, for the larger triangle, the length of the opposite side divided by the length of the adjacent side is

$$\begin{aligned} o_2/a_2 &= (k.o_1)/(k.a_1) \\ &= o_1/a_1 \end{aligned} \tag{5.1}$$

i.e. dividing the length of the opposite side by the length of the adjacent side gives the same value for both triangles. This value will only depend upon the angle θ. This ratio is called the **tangent** of θ (or $\tan(\theta)$) and can be found either by looking it up in tables or by the use of a calculator. Thus,

$$\tan(\theta) = \text{length of the opposite side / length of the adjacent side} \tag{5.2}$$

It is worth having a look at the ratios formed from pairs of sides other than a and o in Fig. 5.4. For example, the length of the opposite side divided by the length of the hypotenuse is also the same for both triangles since

$$o_2/h_2 = (k.o_1)/(k.h_1)$$
$$= o_1/h_1 \qquad (5.3)$$

This ratio, again, only depends upon the angle θ and is called the sine of θ or,

$$\sin(\theta) = \text{length of the opposite side/length of the hypotenuse} \qquad (5.4)$$

Finally, the length of the adjacent side divided by the length of the hypotenuse is a constant for the two triangles since

$$a_2/h_2 = (k.a_1)/(k.h_1)$$
$$= a_1/h_1 \qquad (5.5)$$

This ratio is called the cosine of θ, i.e.

$$\cos(\theta) = \text{length of the adjacent side / length of the hypotenuse} \qquad (5.6)$$

N.B. The definitions of tan, cos and sin given above are only true for right-angled triangles.

> **Question 5.4** The hypotenuse of a right-angled triangle is twice the length of one of the other sides. Calculate cos, sin and tan for the angles in the triangle. (Hint: Let one side have a length x giving a hypotenuse of length $2x$. Then use Pythagoras' theorem (i.e. $h^2 = a^2 + o^2$) to find the length of the third side. You will probably find a sketch helpful.)

What are these functions used for? Figure 5.5 illustrates a common situation in which the sine function can be used. The geological map (Fig. 5.5a) shows an alternating sequence of sandstone and limestone formations. One of the sandstone formations has an outcrop width of 1.25 km and its beds dip at 27°. What is the true thickness of this formation? Figure 5.5b shows how the apparent width, W, of a bed or formation is related to its true thickness, T, and its dip. From the definition of sine (Eqn. 5.4) it follows that

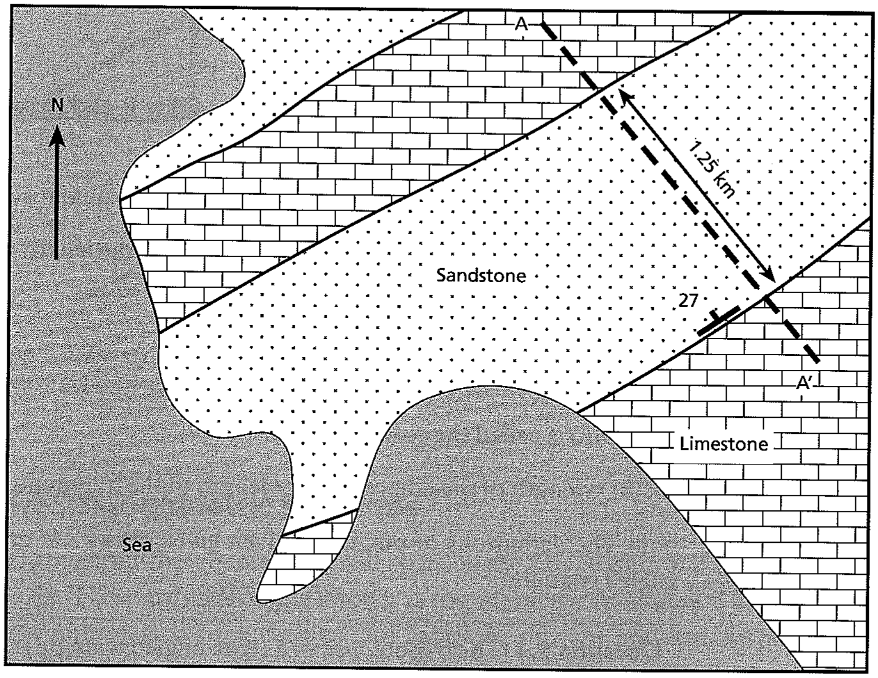
$$\sin(\text{Dip}) = T/W \qquad (5.7)$$

which, after rearrangement, yields

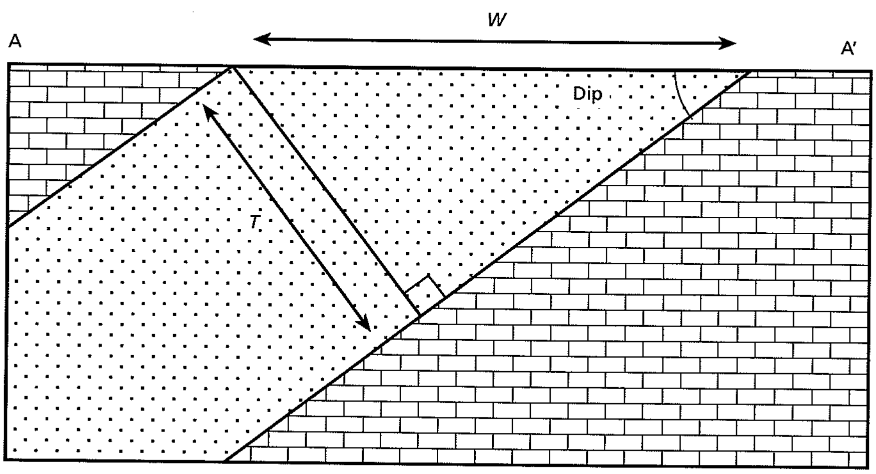
$$T = W \sin(\text{Dip}) \qquad (5.8)$$

Substituting the known values for W and dip and using a calculator (or tables) to calculate sin(Dip), gives

$$T = 1.25 \sin(27°)$$
$$= 1.25 \times 0.454$$
$$= 0.567 \text{ km}$$
$$= 567 \text{ m} \qquad (5.9)$$



(a)



(b)

Fig. 5.5 (a) Geological map showing alternating sandstone and limestone bedding. One of the sandstone formations has a width of 1.25 km and a dip of 27°. (b) Vertical cross-section through a dipping bed which has a true thickness T and an apparent thickness on the surface of W.

Question 5.5 A cliff has a height of 130 m. A particular sedimentary bed outcrops at the cliff top and dips at 42.5° in a direction parallel to the cliff edge. Draw a sketch of this and, by considering the definition of the tangent function, determine how far away, horizontally, the same bed outcrops at the cliff base.

The **inverse trigonometric functions** produce the angle corresponding to a particular value for a sine, cosine or tangent. For example, $\sin(37°) = 0.602$ and therefore the inverse sine of 0.602 equals 37°. The inverse tangent, sine and cosine functions are sometimes called the arctangent, arcsine and arcosine functions. In equations they are denoted by $\tan^{-1}$, $\sin^{-1}$ and $\cos^{-1}$, respectively (e.g. $\sin^{-1}(0.602) = 37°$). Some calculators use a notation of atan, asin and acos instead or, very occasionally, arctan, arcsin and arcos.

The standard notation is very poor since there is a very similar notation for denoting powers of trigonometric functions. For example, the square of $\tan(\theta)$ (i.e. $\tan(\theta).\tan(\theta)$) is usually written $\tan^2(\theta)$. Thus, $\tan^{-1}(\theta)$ might be thought, erroneously, to be the same as $1/\tan(\theta)$. Unfortunately, this way of denoting the inverse trigonometric functions is very well established and is unlikely to be dropped now.

The fact that, with these inverse functions, angles can now be found from knowledge of their sines, cosines or tangents greatly increases the power of trigonometry. For example, the inverse tangent function can be used to determine true bed dips from a cross section which has vertical exaggeration. Geological cross sections frequently have different scales in the vertical and horizontal directions since data may be mapped over several kilometres horizontally but only extrapolated downwards for a few hundred metres. Figure 5.6 shows an example in which the cross section is 4 or 5 km wide but only about 100 m deep. The vertical exaggeration here is about 12 to 1 (i.e. the vertical scale is stretched 12-fold relative to the horizontal scale). Thus, to

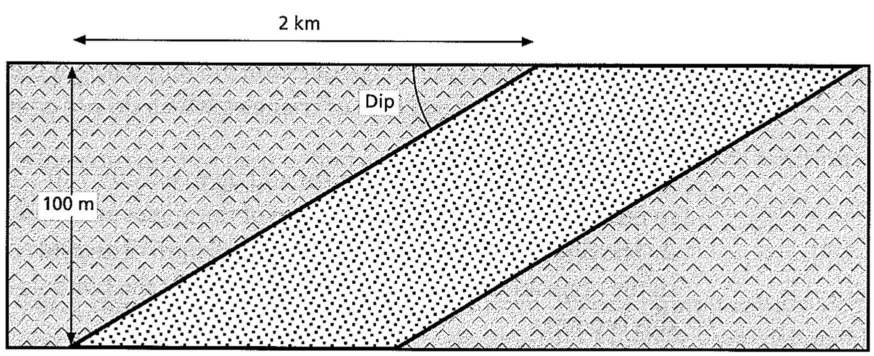


Fig. 5.6 Cross-section in which the vertical scale is about 12 times larger than the horizontal scale. The result is an apparent bed dip which is much greater than the true dip.

get a section in true scale, all vertical distances should be shrunk by a factor of 12. As a result, the beds, which appear to have a dip of about 30°, have a true dip which is much less. The true dip may be found by noting that, from Eqn. 5.2 and Fig. 5.6,

$$\begin{aligned}\tan(\text{dip}) &= \text{opposite/adjacent}\\ &= 100\text{ m}/2\text{ km} = 100/2000\\ &= 0.05\end{aligned} \qquad (5.10)$$

therefore, using the inverse tangent,

$$\begin{aligned}\text{dip} &= \tan^{-1}(0.05)\\ &= 2.86°\end{aligned} \qquad (5.11)$$

Thus, the true dip is less than 3°, i.e. about one-tenth of the apparent dip in Fig. 5.6.

Question 5.6 A cliff has a height of 45 m. A bed at the cliff top outcrops 110 m, horizontally, from where it outcrops at the cliff base. Draw a sketch and determine the value of the tangent of the bed dip. Using the inverse tangent, find the dip in degrees.

5.3 Determining unknown angles and distances

In questions 5.1 and 5.2 earlier in this chapter, the angles and side lengths of several triangles were determined by drawing a sketch using the supplied information and measuring the unknown lengths and angles. Clearly, it would be more convenient and more accurate if the unknowns could be calculated, rather than measured, and this is indeed possible. In fact, the problems and examples discussed in the previous section have been doing precisely this for the special case of triangles containing a right angle. In question 5.6, for example, you were given the lengths of two sides and the value of one angle (i.e. 90°) from which the remaining two angles and one side length could be calculated (although question 5.6 only asks for one angle).

For the more general case of triangles which do not contain a right angle, three rules are needed:

1 The 180° rule. The angles must add up to exactly 180°. This rule allows us to find the third angle whenever two of the angles are known.

2 The sine rule. For a given triangle, the length of any side divided by the sine of the opposite angle is a constant. In terms of the symbols defined in Fig. 5.2 this becomes

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)} \qquad (5.12)$$

3 The cosine rule. This is a generalization of Pythagoras' theorem to cover non-right-angled triangles. In terms of the symbols defined in Fig. 5.2, the cosine rule is

$$a^2 = b^2 + c^2 - 2bc.\cos(A) \tag{5.13}$$

or

$$b^2 = a^2 + c^2 - 2ac.\cos(B) \tag{5.14}$$

or

$$c^2 = a^2 + b^2 - 2ab.\cos(C) \tag{5.15}$$

> **Question 5.7** Using the symbols from Fig. 5.2:
> (i) If $A = B = 70°$, use the 180° rule to find angle C,
> (ii) $b = 3$ km, $c = 2$ km and $C = 40°$, use the sine rule to find angle B,
> (iii) If $b = 3$ km, $c = 1$ km and $A = 37°$, use the cosine rule to find length a.

> **Question 5.8** If angle A is a right angle, show that Eqn. 5.13 reduces to Pythagoras' theorem $a^2 = b^2 + c^2$.

The 180° rule, the sine rule and the cosine rule are used in different ways and different orders depending upon the information known at the start of a specific problem. In general, a triangle is characterized by six quantities (i.e., three lengths and three angles) and all six can be found provided at least one length is known plus any two other pieces of information.

Given the three rules, and three pieces of information, most problems can be solved in several different ways. Suppose, for example, that three sides and zero angles are known. The first step is to use the known lengths to calculate one of the unknown angles. This implies that the cosine rule should be used since the other rules all involve more than one angle. Equation 5.13 can be rearranged into

$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc} \tag{5.16}$$

which can then be used to find angle A by using the inverse cosine function to give

$$A = \cos^{-1}[(b^2 + c^2 - a^2)/2bc] \tag{5.17}$$

Three sides and one angle, A, are now known. At this point it is possible to proceed using either the cosine rule or the sine rule. However, where possible and for reasons given later, it is usually best to avoid using the sine rule. Hence, rearranging Eqn. 5.14 gives

$$\cos(B) = \frac{a^2 + c^2 - b^2}{2ac} \tag{5.18}$$

and then, using the inverse cosine function

$$B = \cos^{-1}[(a^2 + c^2 - b^2)/2ac] \tag{5.19}$$

The final unknown quantity is the angle C which, using the 180° rule, is found from

$$C = 180 - A - B \tag{5.20}$$

Question 5.9 Two exposures, 500 m apart, are 400 m and 200 m, respectively, from a church. Calculate, using Eqns. 5.17, 5.19 and 5.20, the angles contained by the triangle defined by the two exposures and the church.

Question 5.10 Find the unknown quantities in the following:
(i) $A = 40°, b = 5$ km, $c = 2$ km;
(ii) $A = 40°, b = 3$ km, $a = 2$ km;
(iii) $A = 40°, B = 60°, a = 3$ km.

5.4 Cartesian coordinates and trigonometric functions of angles bigger than 90°

The definitions of the trigonometric functions given in Section 5.2 are only valid for angles less than 90°. However, triangles frequently have one angle greater than this. So, how are these functions defined in these cases? It is easiest to begin by first discussing **Cartesian coordinates** (Fig. 5.7). This is a way of specifying any location in a plane by giving the horizontal and vertical distance from an **origin** (the centre of the coordinate system where $x = y = 0$). Point A, for example, is at the location $x = 5$, $y = 10$. This is frequently abbreviated to 'the point (5,10)'. Note that points to the left of the origin have a negative x coordinate and points below the origin have a negative y coordinate.

This same coordinate system could be used to specify angles by drawing lines between the origin and these points (Fig. 5.8). The corresponding angle is that formed between the x-axis and the line, measured in an anticlockwise sense. (N.B. When measuring compass bearings, angles are measured clockwise around from North. It is unfortunate that mathematicians and cartographers have settled on different conventions but you will have to get used to using these in different contexts.)

(5.18)

(5.19)

g the 180° rule, is

(5.20)

d 200 m, respect-
19 and 5.20, the
xposures and the

ving:

: functions of

ection 5.2 are only
tly have one angle
ese cases? It is easi-
5.7). This is a way
al and vertical dis-
where $x = y = 0$).
This is frequently
e left of the origin
have a negative y

angles by drawing
rresponding angle
d in an anticlock-
gles are measured
athematicians and
ou will have to get

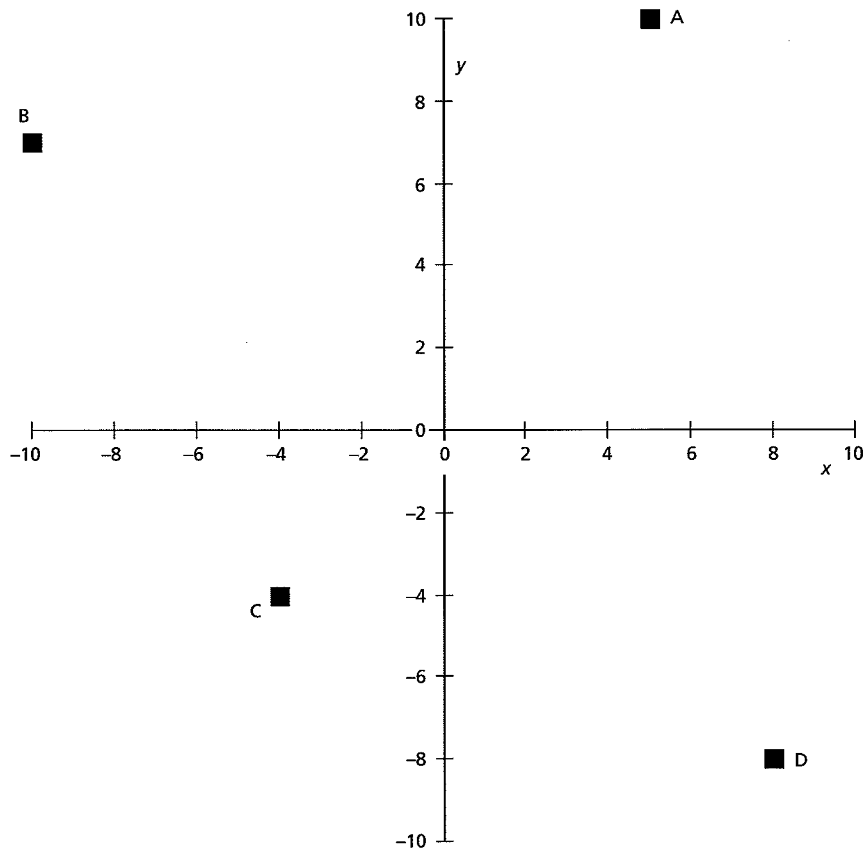


Fig. 5.7 Cartesian coordinates to specify locations of points. For example point A is at $x = 5$, $y = 10$ and point C is at $x = -4$, $y = -4$.

Starting with the point A: the angle, θ_a, corresponding to the point (5,10) has a tangent of

$$\begin{aligned}\tan(\theta_a) &= \text{opposite/adjacent}\\ &= y\text{-coordinate}/x\text{-coordinate}\\ &= 10/5\\ &= 2.0 \end{aligned} \quad (5.21)$$

i.e. the angle is $\tan^{-1}(2) = 63.4°$. Now, the same procedure could be attempted with point B at (−10,7). The angle of interest is now between 90° and 180°. Note that x is therefore negative. In other words, the length of the adjacent side is negative giving

$$\begin{aligned}\tan(\theta_b) &= \text{opposite/adjacent}\\ &= 7/-10\\ &= -0.7 \end{aligned} \quad (5.22)$$

Thus, for this case the tangent is a negative number.

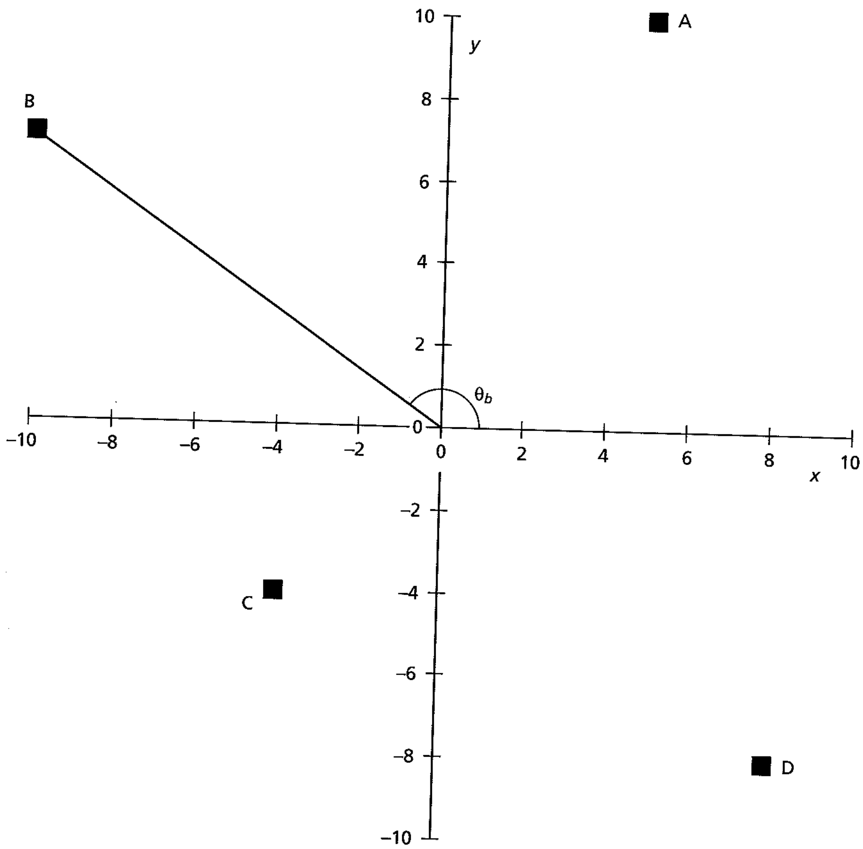


Fig. 5.8 Using cartesian coodinates to specify angles. Angles are defined anti-clockwise from the x-axis to each of the lines. For clarity, only one angle is shown.

> **Question 5.11** Repeat the above procedure to find the tangents of the angles produced using points C and D in Fig. 5.8.

The same process could now be attempted for the cosines and sines of the angles produced by points A, B, C and D. For the sine and cosine calculations, the hypotenuse is the line from the origin to the point and is always taken to have a positive length. Figure 5.9 summarizes thé results by displaying sine, cosine and tangent for angles between 0° and 360°.

Note that there is more than one angle which gives rise to any particular value for the sine, cosine or tangent (e.g. tan(45°) = tan(225°) = 1.0). For this reason, the angles obtained from calculating the inverse trigonometric functions are not unique. Thus, your calculator would give $\tan^{-1}(1.0) = 45°$ but the answer could be 225°. In general, there are two angles between 0° and 360° which give rise to any given value for sine, cosine or tangent. You

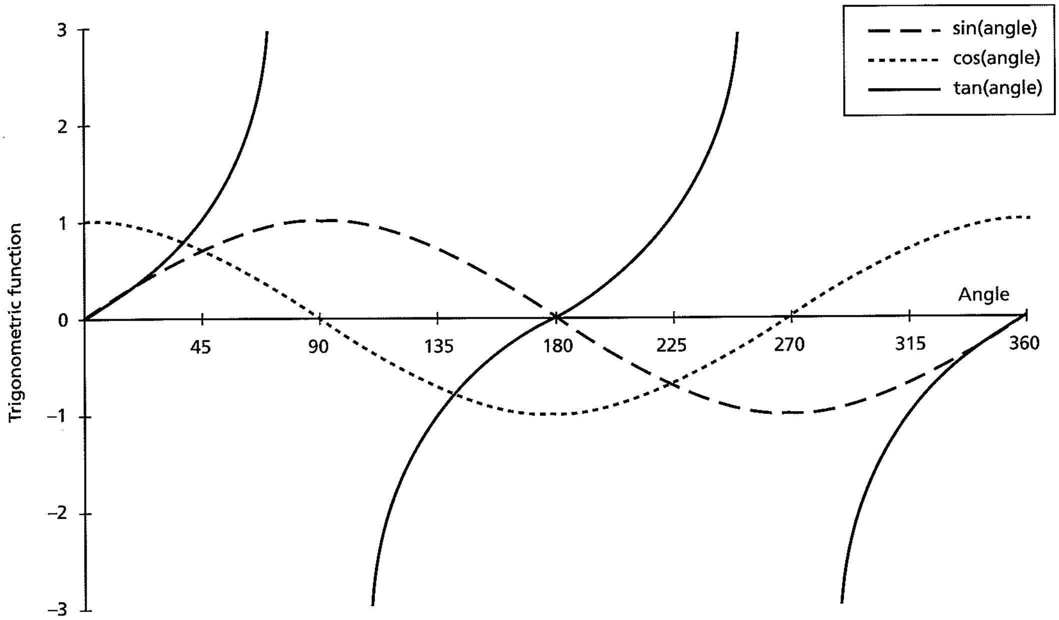


Fig. 5.9 The sine, cosine and tangent functions for angles between 0° and 360°.

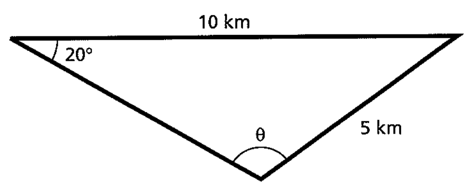


Fig. 5.10 What size is θ in this triangle?

should be aware that this non-uniqueness can, occasionally, result in incorrect answers. Consider Fig. 5.10 in which a triangle is shown with an unknown angle, θ, clearly much larger than 90°.

The obvious way to determine θ is to use the sine rule which leads to

$$\sin(20)/5 = \sin(\theta)/10 \tag{5.23}$$

thus

$$\begin{aligned}\sin(\theta) &= 2\sin(20)\\ &= 0.684\end{aligned} \tag{5.24}$$

giving

$$\begin{aligned}\theta &= \sin^{-1}(0.684)\\ &= 43.2°\end{aligned} \tag{5.25}$$

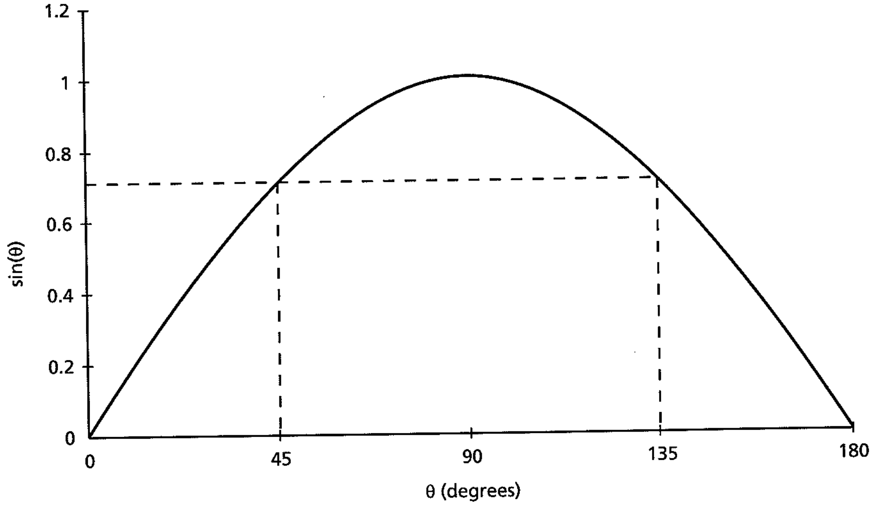


Fig. 5.11 Non-uniqueness of the inverse sine function. For Example $\sin^{-1}(0.684) = 43.2°$ or 136.8° (i.e. 180−43.2).

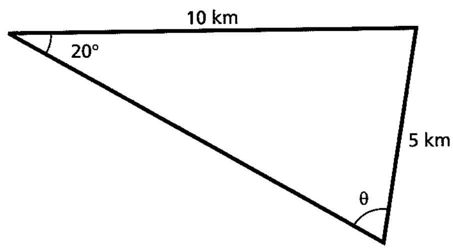


Fig. 5.12 Compare this to Fig. 5.10. The information specified is precisely the same but, this time, a solution less than 90° is plausible.

which is plainly wrong. The reason for this is that there are two angles between 0° and 180° which have a sine of 0.684. Inspecting Fig. 5.11, an angle of 180 − 43.2 degrees has the same sine as 43.2°. Thus, the correct answer is 136.8°.

The reason for this uncertainty is simply that the other answer, 43.2°, is possible with the information given. This is illustrated by Fig. 5.12 which has exactly the same known starting information (2 sides and one opposite angle) but which actually has a solution of 43.2°. In the case of two sides and one opposite angle, we must also know if the other opposite angle is less than or greater than 90°.

In conclusion, whenever you use the inverse sine, cosine or tangent functions, calculate both solutions and then decide which is appropriate in the particular case you are investigating. Note that, for the inverse cosine and inverse tangent functions, the larger of the two solutions is greater than 180° and, therefore, can be ignored if the answer is the angle of a triangle.

However, in some other types of problem, not discussed in this book, these large solutions may be valid. The two solutions for the inverse sine function, on the other hand, are both less than 180° and therefore are both plausible. This is the reason that, earlier, I suggested avoiding the sine rule whenever possible.

5.5 Trigonometry in a three-dimensional world

Up to now, I have used trigonometry purely in two dimensions. However, geology is a three-dimensional subject. Figure 5.13a illustrates a typical three-dimensional problem. Imagine a dipping bedding plane outcropping on a cliff face which is not parallel to the direction of maximum slope. This will result in an apparent dip, on the cliff face, which is less than the true dip. The most extreme case is where the cliff face is at right angles to the direction of dip (i.e. the cliff is in the *strike* direction). In this extreme case the apparent dip of the beds is zero! Is there a simple relationship between the true dip and the apparent dip?

Figure 5.13b shows a construction for determining this relationship. In this diagram the bedding plane has a true dip, θ, in a direction parallel to the x-axis. The line OC represents the direction along which the bedding plane is cut (i.e. the cliff face). This direction is at an angle α to the dip direction. This results in an apparent dip of θ'. Note that the angles COA, BOA and OBC are all right angles. From this it follows that

$$\tan(\theta) = OA/OB \tag{5.26}$$

$$\tan(\theta') = OA/OC \tag{5.27}$$

and

$$\cos(\alpha) = OB/OC \tag{5.28}$$

Equation 5.27 and 5.28 can be combined as follows

$$\begin{aligned}\tan(\theta') &= OA/OC = (OA/OB).(OB/OC)\\ &= (OA/OB)\cos(\alpha)\end{aligned} \tag{5.29}$$

which, together with Eqn. 5.26, leads to

$$\tan(\theta') = \tan(\theta)\cos(\alpha) \tag{5.30}$$

which relates the apparent dip, θ', to the true dip, θ, and the angle, α, which the cliff makes to the dip direction. From this, the apparent dip can be found using

$$\theta' = \tan^{-1}[\tan(\theta)\cos(\alpha)] \tag{5.31}$$

or the true dip can be calculated from the apparent dip using

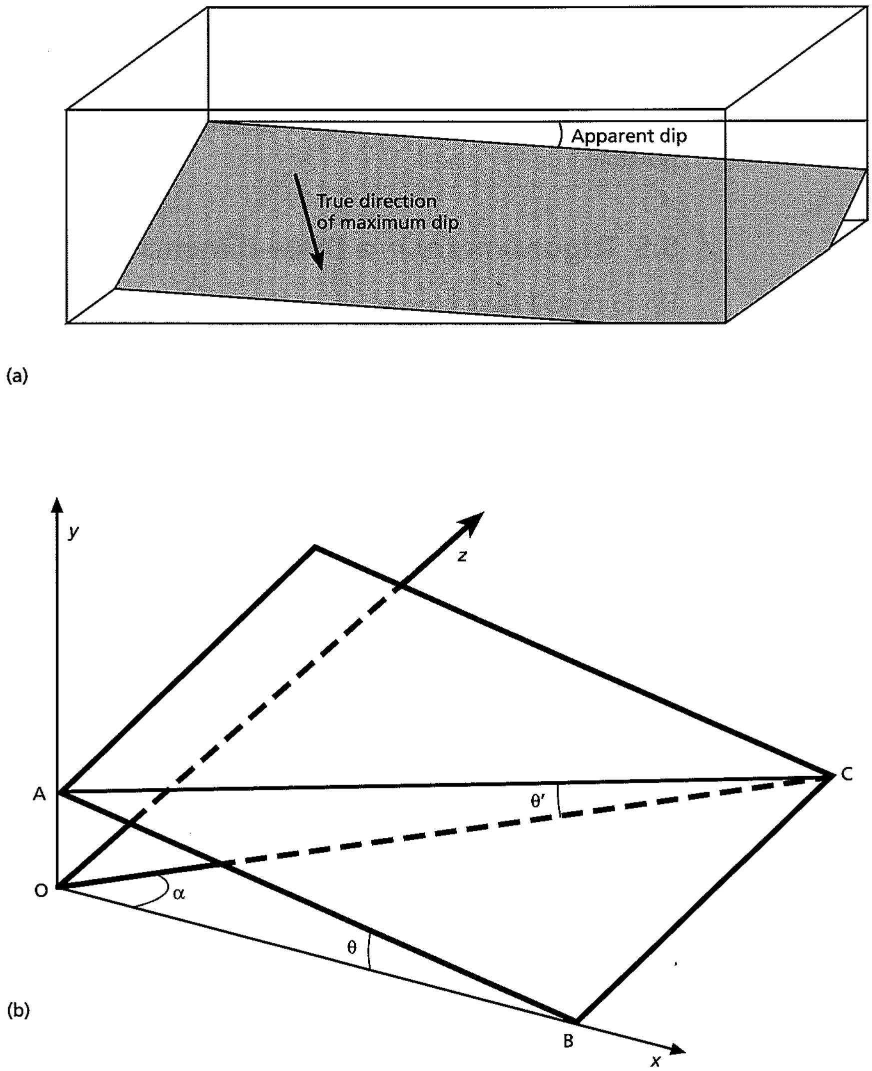


Fig. 5.13 (a) A bedding plane dipping in a direction not parallel to a cliff (represented here by the back face of the cuboid). This results in an apparent dip on the cliff face which is less than the true dip. (b) Construction for determining the true dip from the apparent dip. Triangle AOB is parallel to the true dip direction whilst AOC is parallel to the cliff face.

$$\theta = \tan^{-1}[\tan(\theta')/\cos(\alpha)] \quad (5.32)$$

If the apparent dip is 32°, measured in a direction 25° from the direction of maximum dip, the true dip is

$$\begin{aligned}\theta &= \tan^{-1}[\tan(32)/\cos(25)] \\ &= \tan^{-1}[0.625/0.906] \\ &= \tan^{-1}[0.689] \\ &= 34.6° \end{aligned} \quad (5.33)$$

> **Question 5.12** On a cliff face, the apparent dip is 25° whilst the true dip is 35°. What is the angle between the cliff face and the strike direction?

Finding the true dip and its direction, in the field, can become even more complex than indicated. Additional difficulties not considered so far are such things as the effect of uneven topography, non-planar bedding and measurements made on inclined surfaces. Fortunately, there is a much easier approach using **stereographic projection**, a subject that will be introduced in the next chapter.

5.6 Introduction to vectors

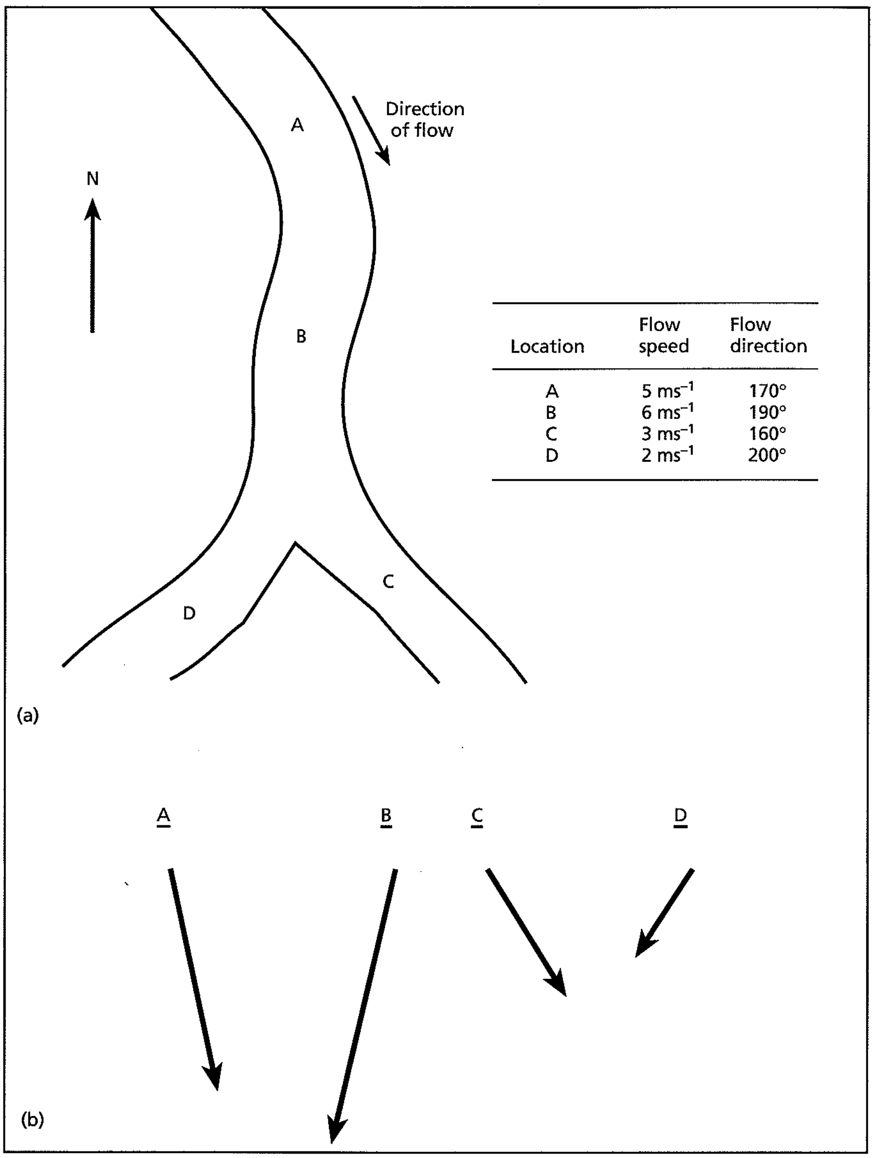
This section will give a brief introduction to **vectors**. A more detailed treatment is beyond the scope of this book, but the most common vector operation (**vector addition**) is covered.

A vector is any quantity which has a direction as well as a magnitude. River channels, for example, can be described in terms of their direction and rate of flow. Another example is the Earth's magnetic field which, at any given point on the Earth's surface, has a definite direction (roughly speaking the field points North with a dip which depends upon latitude) as well as a definite strength (the strength increases towards the poles). Quantities which only have magnitude but no direction are called **scalars** (e.g. temperature).

> **Question 5.13** Are the following vector or scalar quantities?
> (i) Mass;
> (ii) Gravitational acceleration;
> (iii) Age;
> (iv) The line joining an exposure location to a church.

Figure 5.14 shows a series of vectors representing flow at various locations on a river. The arrows point in the flow direction and have a length which is proportional to the flow speed. These arrows are diagrammatic representations of the flow vectors. These vectors can be denoted by the letters $\underline{A}$, $\underline{B}$, $\underline{C}$ and $\underline{D}$ where underlining is a way of indicating that they are vectors. An alternative notation is to indicate vectors by using boldface (i.e. **A**, **B**, **C** and **D**). In this section, I shall deliberately alternate these so that you get used to seeing vectors written both ways. Obviously, if you are writing vector expressions by hand, it is easiest to use the underlining convention.

An important property of vectors is that they may be added together. Vector addition is simply the process of combining the vectors nose-to-tail



Location	Flow speed	Flow direction
A	5 ms^{-1}	170°
B	6 ms^{-1}	190°
C	3 ms^{-1}	160°
D	2 ms^{-1}	200°

Fig. 5.14 (a) A river flowing, roughly, southwards. At points A, B, C and D the river speed and direction are as shown in the table. (b) Vector representation of the river flow at A, B, C and D. The direction of the arrows shows the flow direction and their length is proportional to the speed.

as shown in Fig. 5.15. The resultant vector is obtained by drawing a vector from the tail of the first vector to the nose of the last. This operation can be algebraically represented by the equation

$$\underline{r} = \underline{a} + \underline{b} + \underline{c} + \underline{d} \tag{5.34}$$

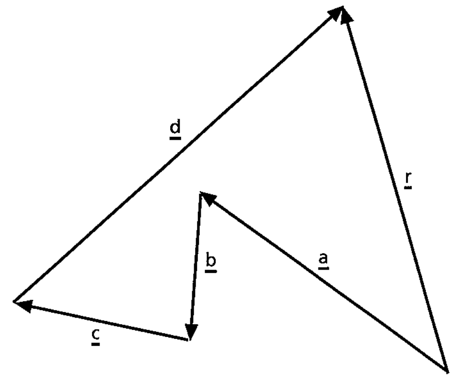


Fig. 5.15 Vector addition. Vectors a, b, c and d are added nose-to-tail as shown to give the result r.

An important point about this addition is that it is not the same as adding the vector lengths and vector directions separately.

> **Question 5.14** Draw a set of x–y axes on a sheet of paper. Then draw the vectors:
> (i) Vector a: Length 3 cm, direction 10° clockwise from the x-axis;
> (ii) Vector b: Length 5 cm, direction 50° clockwise from the x-axis;
> (iii) Vector c: Length 3 cm, direction 190° clockwise from the x-axis;
> (iv) The vector d = a + b;
> (v) The vector e = a + c.

Another useful operation on vectors is **scalar multiplication** (don't confuse this with the **scalar product,** a more sophisticated vector operation, which will not be discussed further here). In scalar multiplication the vector length is simply increased by multiplying it by a scalar. Thus, a vector in the direction 13° E of N with a magnitude of 5 km becomes, after scalar multiplication by 3, a vector in the same direction (i.e. 13° E of N) but with a magnitude increased to 15 km. A small complication is the effect of multiplying by a negative quantity (e.g. –3). In this case, the direction of the vector is reversed and so multiplying a vector in the direction 13° E of N with a magnitude of 5 km becomes, after scalar multiplication by –1, a vector in the direction 193° E of N with a magnitude of 5 km.

Vector addition and scalar multiplication together allow a new way of specifying a vector to be introduced. The idea is to specify the vector in terms of the lengths of **component vectors** in the x and y directions. This is illustrated in Fig. 5.16. The x-component and the y-component sum to produce the given vector. The vector can then be written down as

$$\mathbf{a} = x\mathbf{i} + y\mathbf{j} \quad (5.35)$$

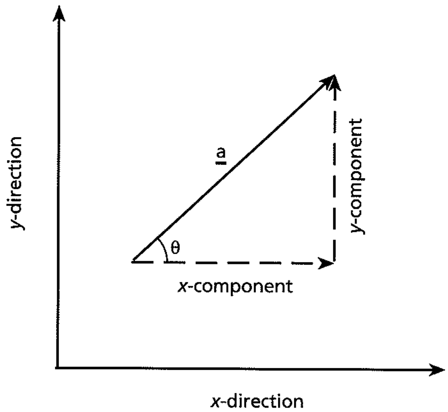


Fig. 5.16 Any vector can be thought of as the result of summing an x-component vector and a y-component vector.

where **i** and **j** are vectors of length 1.0 in the x and y directions, respectively, and where x and y are the lengths of the x and y components. Thus, $x\mathbf{i}$ is a vector of length x in the x-direction and $y\mathbf{j}$ is a vector of length y in the y-direction. In other words, vector **a** is the sum of the x and y component vectors. Vectors such as **i** and **j**, which are of unit length, are known as **unit vectors.**

To convert one form of vector specification into the other, it is only necessary to use a little trigonometry. From Fig. 5.16, the lengths of the x and y components are

$$x = a \cos(\theta) \tag{5.36}$$

and

$$y = a \sin(\theta) \tag{5.37}$$

where a is the length of vector **a** and θ is the angle which vector **a** makes to the x-direction. Thus, if the length, a, and direction, θ, of the vector are known, the x and y components can be easily found. To convert from components to vector magnitude, Pythagoras' theorem gives

$$\text{vector length, } a = \sqrt{x^2 + y^2} \tag{5.38}$$

The vector direction follows from the definition of the tangent function and, for the case of Fig. 5.16, gives

$$\text{vector direction, } \theta = \tan^{-1}(y/x) \tag{5.39}$$

Question 5.15 Use the definitions of the sine and cosine functions and Fig. 5.16 to derive Eqns. 5.36 and 5.37 above.

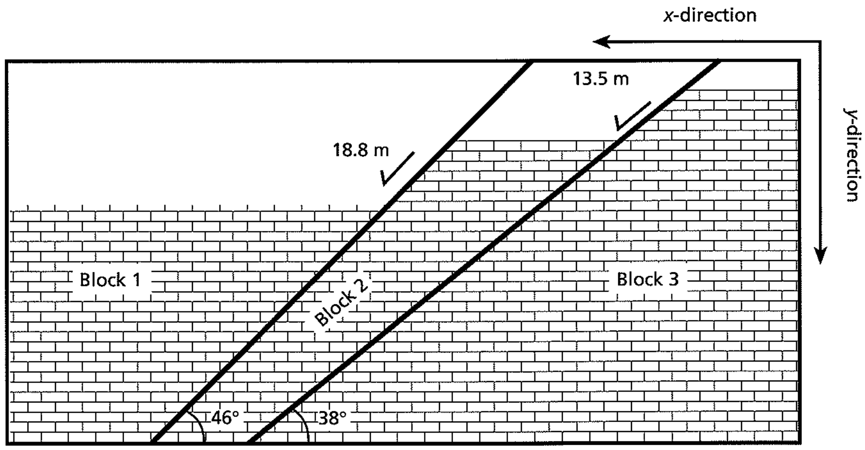


Fig. 5.17 Slip vectors for two faults. The overall slip of block 1 relative to block 3 can be obtained by vector addition of these two slip vectors.

The advantage of recasting vectors in terms of their components is that it makes vector addition much simpler. To add vectors together, you simply add the components. The example illustrated in Fig. 5.17 should make everything much clearer. In this example there are two closely spaced faults with different throws and different fault dips. The question is: what is the total movement of block 1 relative to block 3? Using vectors makes this problem extremely straightforward. All we do is add the slip vector (i.e. a vector representing the direction of slip and amount of throw) for fault 1 to the slip vector for fault 2. So, what are the slip vectors, $\mathbf{s}_1$ and $\mathbf{s}_2$, for each of the faults? Using trigonometry in an identical manner to that used for determining Eqns. 5.36 and 5.37 leads to

$$\begin{aligned}\mathbf{s}_1 &= 18.8\cos(46°)\mathbf{i} + 18.8\sin(46°)\mathbf{k}\\ &= 13.1\mathbf{i} + 13.5\mathbf{k}\end{aligned} \tag{5.40}$$

and

$$\begin{aligned}\mathbf{s}_2 &= 13.5\cos(38°)\mathbf{i} + 13.5\sin(38°)\mathbf{k}\\ &= 10.6\mathbf{i} + 8.3\mathbf{k}\end{aligned} \tag{5.41}$$

giving a resultant, total slip, of

$$\begin{aligned}\mathbf{s} &= \mathbf{s}_1 + \mathbf{s}_2\\ &= (13.1 + 10.6)\mathbf{i} + (13.5 + 8.3)\mathbf{k}\\ &= 23.7\mathbf{i} + 21.8\mathbf{k}\end{aligned} \tag{5.42}$$

However, we would probably wish to have the final answer in the form of the dip and throw of a single fault which would give the same effect. Thus, we need to convert back from vector components to vector direction and magnitude. Applying the same principles as used in Eqns. 5.38 and 5.39 to the fault throw problem gives

$$\text{Total throw} = \sqrt{23.7^2 + 21.8^2} = 32.2 \text{ m} \tag{5.43}$$

and

$$\begin{aligned}\text{Equivalent single fault dip} &= \tan^{-1}(21.8/23.7)\\ &= 42.6°\end{aligned} \tag{5.44}$$

Thus, a single fault dipping at 42.6° with a throw of 32.2 m, would have given an identical vertical and horizontal movement to block 1 relative to block 3. The resultant dip direction is called a **vector mean** direction. This kind of analysis would be useful when examining the variation in extension across fault systems as we move along strike. The number, dip and throw of faults generally varies along strike but by summing the individual slip vectors we could directly compare the size and direction of extension. In general, this analysis would need to be performed as a problem in three dimensions, in which case the slip vectors have three components rather than two. However, apart from this, the methods used would be identical.

Question 5.16 Three adjacent faults have throws and dips of: (i) 10 m at 60°; (ii) 5 m at 65°; (iii) 12 m at 45°. Calculate the total slip vector.

5.7 Further questions

5.17 Evaluate:
(i) $\cos(15°)$; (ii) $\sin(1.2 \text{ radians})$; (iii) $\tan^{-1}(0.5)$; (iv) $\cos^2(27°)$;
(v) $(\tan(0.5°))^{-1}$

5.18 If the Earth were a perfect sphere, show that the radius, r, of a circle of latitude is given by

$$r = R\cos(\phi)$$

where R is the Earth's radius and ϕ is the latitude.

5.19 An alluvial fan slopes at an angle of 5° to the horizontal and the distance from the fan origin to its base (measured along the fan surface) is 5 km. Calculate the height of the fan origin above its base.

5.20 Look at Fig. 5.18 which shows a geological map (Fig. 5.18a) and a section (Fig. 5.18b) drawn from a cliff at the location shown on the map. The map indicates that in the area of the section the direction of strike is 72° E of

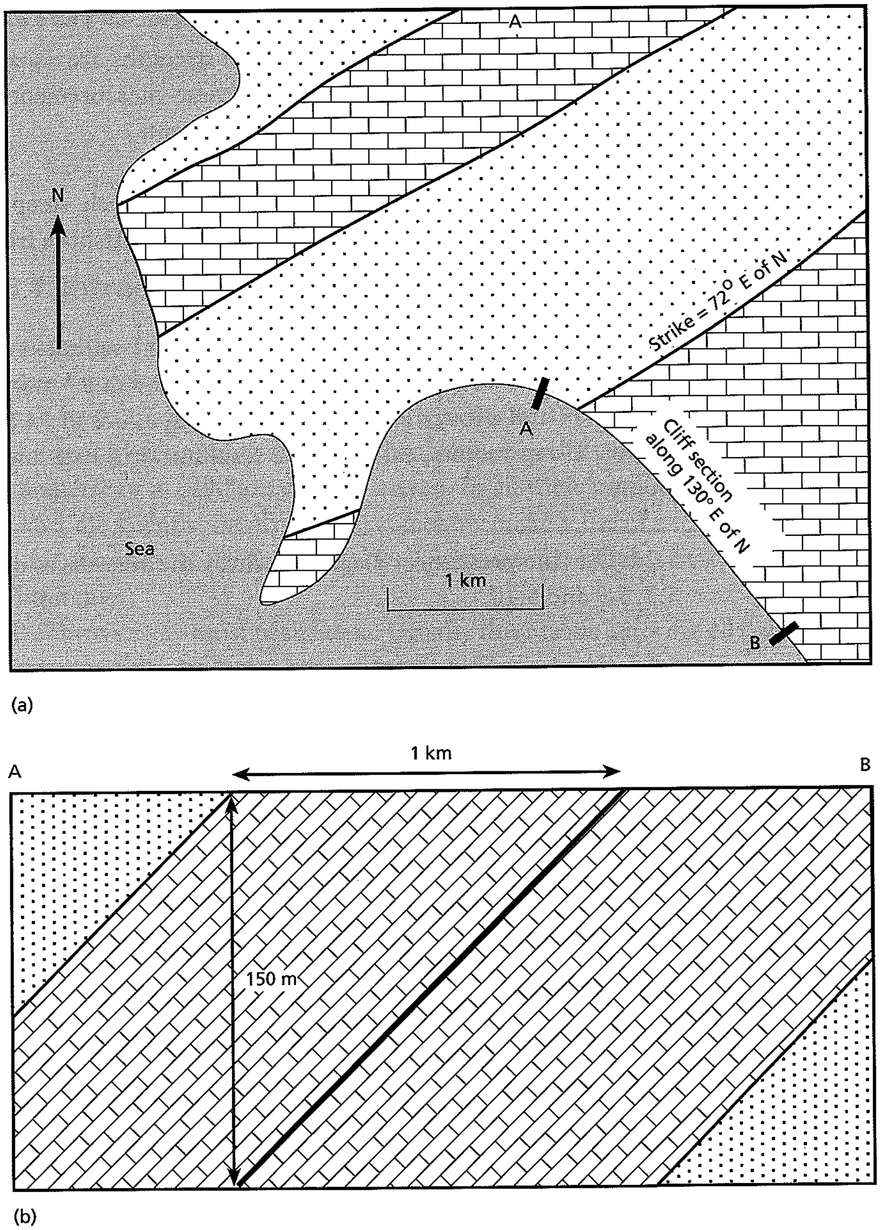


Fig. 5.18 (a) Geological map showing the position and alignment of the section shown in (b) and the strike direction of the beds.

N and the section itself is aligned along 130° E of N. From the information shown on the section, determine the true dip of the beds.

5.21 Remanent magnetism of 10 specimens collected from a Tertiary sill had the following azimuthal directions in degrees E of N:

331, 5, 347, 351, 3, 342, 338, 355, 349, 17.
Assuming that the remanent magnetism strengths are equal (say unity), calculate the vector mean direction for these measurements. Do this either graphically or by calculating vector components.

5.22 Carbonate platform foreslopes can be much steeper than those of deltas. If the water depth is 100 m only 500 m offshore from the slope top, what is the slope?

5.23 A river plume is transporting suspended sediment southwards at a rate of 0.1 m s^{-1}. However, a tidal current of 0.5 m s^{-1} moving in a direction 60° E of N and a longshore drift moving west at 0.2 m s^{-1} are superimposed on this. Using vectors, calculate the resultant drift rate and direction of the plume.

5.24 Use the spreadsheet *Trig.xls* to check the answers to questions 5.1, 5.2, 5.7, 5.9 and 5.10.

5.25 Use the spreadsheet *Vsum.xls* to check the answers to questions 5.16 and 5.21.