1 Mathematics as a tool for solving geological problems

1.1 Introduction

This book is not about specialized geological mathematics. Mostly, this book is about simple mathematics, the sort that many people are introduced to at school. However, such mathematics is frequently poorly understood by geology undergraduates and few students are able to use the maths they know for solving realistic problems. The objectives of this book are to improve understanding of simple mathematics through the use of geological examples and to improve ability to apply mathematics to geological problems.

This is not a formal mathematics textbook. My aim is to try to instil an intuitive feel for maths. I believe that this is more helpful than a rigid, formal treatment since formality can often obscure the underlying simplicity of the ideas.

Although this book concentrates upon standard mathematical procedures, it does contain a few more specialized techniques. The majority of the mathematics encountered by typical undergraduate students is therefore covered here. The exception is, perhaps, statistics which forms a large part of geomathematics and which is well covered by many excellent textbooks. The statistics chapter in this book should form a good introduction to the material covered in those more specialized texts.

Mathematics is much more akin to a language than a science. It is a method of communication rather than a body of knowledge. Thus, the best way to approach a book like this is as you would a text on, say, French or German. You are learning how to communicate with people who understand the mathematical language. You are not learning a collection of facts. Another similarity to learning a language is that you must never pass on to the next lesson until you have grasped the current one. If you do, you will get hopelessly lost and demoralized since subsequent chapters will simply make no sense.

So that you know you have understood sufficiently to move on, each chapter is sprinkled with examples for you to attempt. Mostly these are very short and simple. A few, however, are more difficult and are designed to make you think carefully about the maths just discussed. If you are unable to do one, you should read over the preceding paragraphs again and make sure that you have understood everything. If that does not help then get assistance. Each chapter concludes with additional simple questions as well as more wide ranging questions which will test your ability to apply what you have learnt to more realistic problems. Outline answers to most questions are given at the end of the book and more complete answers are given for some of the more difficult problems. Look carefully at these complete answers since they also show how your answers should be set out. I assume, throughout this book, that you have a calculator and know how to use it.

One difficulty that many students have with mathematics is the large number of specialized mathematical words. Sometimes these words are completely new to the student whilst other times they are used in a similar, but somewhat more precise, manner to their everyday meaning. It is impossible to avoid use of such words since they are vital in mathematics. Wherever I introduce such a word it is in bold face (e.g. **jargon**).

This first chapter is about basic tools that are needed in succeeding chapters and will introduce you to the most important ideas needed for application of mathematical principles to geological problems.

1.2 Mathematics as an approximation to reality

Geology is frequently regarded as a **qualitative** (i.e. descriptive) science. Geological discussions often revolve around questions about what happened and in what order. For example, was a particular area under the sea when a given sedimentary rock was deposited and does the erosive surface at the top imply that uplift above sea level occurred subsequently? However, the same geological information can be described **quantitatively** (i.e. by numbers). In the preceding example, how deep was the sea and how long was it before uplift occurred? Geology is also concerned with the influence of one process upon another. How does changing water depth affect sediment type? Once again, it is possible to do this quantitatively by producing equations relating, say, grain size to water depth (unlikely to be very accurate but in principle possible).

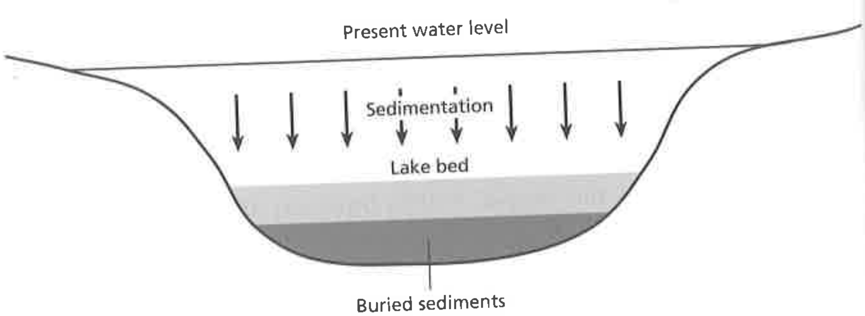

Fig. 1.1 Sedimentation at a lake floor. Older sediments are slowly buried by younger deposits.

Figure 1.1 illustrates a situation in which a quantitative description can be attempted. The figure shows a lake within which sediment, suspended in the water, rains down and slowly builds up on the lake floor. Obviously, early deposits will be covered by later ones. This results in a relationship between depth below lake bed and time since deposition; the deeper you go the older the sediments get. Now, if the rate at which sediments settle upon the floor is approximately constant, sediments buried 2 metres below the lake bed are twice as old as sediments buried by 1 metre and sediments buried by 3 metres are three times as old and so on. Thus, if you double depth, you double the age, if you triple depth you triple the age and so on. This means that the sediment age is **proportional** to burial depth. This can be expressed, mathematically, by the equation

$$Age = k.Depth \tag{1.1}$$

where k is a **constant.** Constants are values which don't change within a given problem. The period symbol, '.', is often used in place of '×' to indicate multiplication and so Eqn. (1.1) reads '*Age* equals k multiplied by *depth*' which can equally well be written in the forms

$$Age = k \times Depth \tag{1.1}$$

or

$$Age = k\,Depth \tag{1.1}$$

All of these different forms for Eqn. 1.1 simply indicate that the age of the sediment equals its depth multiplied by a constant. This constant tells us how rapidly sediments accumulate. A large value for k implies that age increases very rapidly as depth increases (i.e. sediments accumulated very slowly). A low value implies that the age increases more slowly (i.e. sediments accumulated more rapidly). In a particular lake it might take 1500 years for each metre of sediment to accumulate. In this case $k = 1500$ years/metre. A lake with a lower sedimentation rate of, say, 3000 years/metre, would have a more rapid increase in age with depth of burial.

> **Question 1.1** If $k = 1500$ years/metre calculate, using Eqn. 1.1, the age of sediments at depths of 1 metre, 2 metres and 5.3 metres. Repeat the calculations for $k = 3000$ years/metre.

As you see, it is possible to produce mathematical expressions relating geological **variables** to each other. A variable is a quantity which, in a particular problem, can change its value, e.g. the variable called *Age* in Eqn. 1.1 changes when the variable called *Depth* is altered. Are such quantitative descriptions worth bothering with and are the results worth having? Well, it depends! Sometimes such an exercise will not tell you anything you didn't already know. On other occasions the ability to manipulate and combine mathematical expressions can lead to new insight into geological processes. Mathematical expressions also have the great merit of consistency, they always give the same answer when you use the same data (unlike some geologists I know). Finally, mathematical expressions are capable of being definitively tested. An expression can be used to predict a result and that result can then be checked. Equation 1.1 could be used to predict the age at a particular depth and that age could be tested using, say, a geochemical dating method. If the age is very wrong then there is something wrong with the geological or mathematical model, e.g. some important factor is missing.

Unfortunately, it is quite possible for mathematics to give the wrong answer. In fact, mathematical results are rarely 100% correct. Hopefully though, a mathematical relationship is at least approximately true. The lake sediment example, Eqn. 1.1, is a good case. The assumption that sedimentation happened at the same rate throughout deposition and the, hidden, assumption that the sediments are not compacted by the weight of overlying deposits are unlikely to be completely true. However, provided the sedimentation rate does not vary too much and provided sediment compaction is not too extreme, Eqn. 1.1 should be approximately correct. This is all that is necessary for a mathematical formulation to be useful. It is worth keeping this fact, that mathematical expressions are usually approximations, at the back of your mind. People often make the assumption that, because a mathematical expression is being used, the answer must be right. This is simply not true, not even in physics (equations in physics are also approximations to reality although the approximation is usually so good that this can be safely overlooked).

A final general point about using mathematics for solving problems. If you look on any page in this book, or at any mathematical paper in a geological journal, you will see that there is far more text than there is mathematics. A frequent failing in students' use of mathematics is to write down lots of equations with no explanation of what they are or what they mean. The result is an obscure piece of work which nobody else, even the students themselves 6 months later, can understand. It is also a recipe for sloppy or illogical mathematics. A good guide is that there should be rather more English in a piece of mathematics than equations. The object is to tie the mathematics in with the 'real world' it is describing. We are dealing with applied rather than pure mathematics and it is vital that the geological relevance of any mathematics you use is made totally clear. Hence, the aim is to describe the geological context rather than details of the mathematics itself. At the very least, you must describe all of the constants and variables that you use. It is also good practice to number all equations since this makes it easier for yourself, or anybody else, to refer to a particular expression in later discussion. Finally, a clear diagram is usually an important part of a good mathematical explanation.

1.3 Using symbols to represent quantities

The lake sediment equation used the word *Depth* to represent the quantity depth. However, any other symbol would do equally well. Equation 1.1

$$Age = k.Depth \tag{1.1}$$

could be written

$$a = kz \tag{1.2}$$

where a is age, k is the sedimentation constant and z is depth. Alternatively, it could be written

$$\alpha = \kappa\zeta \tag{1.3}$$

where α is age, κ is the sedimentation constant and ζ is depth (see Table 1.1 for a list of Greek letters such as those used here). The point is that it really

Table 1.1 Lower case and upper case letters of the Greek alphabet.

Greek characters	Name
α, Α	alpha
β, Β	beta
γ, Γ	gamma
δ, Δ	delta
ε, Ε	epsilon
ζ, Ζ	zeta
η, Η	eta
θ, Θ	theta
ι, Ι	iota
κ, Κ	kappa
λ, Λ	lambda
μ, Μ	mu
ν, Ν	nu
ξ, Ξ	xi
ο, Ο	omicron
π, Π	pi
ρ, Ρ	rho
σ, Σ	sigma
τ, Τ	tau
υ, Υ	upsilon
φ, Φ	phi
χ, Χ	chi
ψ, Ψ	psi
ω, Ω	omega

Table 1.2 Commonly used symbols and their usual meanings.

Symbol	Usual meaning
z	Depth
T	Temperature
t	Time
x	Horizontal distance
ρ	Density
ϕ	Porosity or grain size
θ	An angle
P	Pressure
r	Radius
v	Velocity
σ	Stress

doesn't matter. Equation 1.3 is just as valid and just as simple as Eqn. 1.1. However, the unfamiliarity of Greek letters can make an equation look rather daunting. The same equation could equally well be written using Hebrew characters, Chinese pictograms, Egyptian hieroglyphics or even some completely new set of symbols. The use of Greek letters extensively throughout mathematics is simply a tradition, although it does have the benefit of doubling the number of symbols available.

Also traditional is the use of particular symbols to represent commonly encountered quantities. A good example is z which is nearly always used to represent depth. A few other common examples are given in Table 1.2 which is far from complete but it should give the general idea. These symbols are so commonly used for these particular variables that people frequently forget to define them. Thus if, in a particular book or paper, the author is discussing crustal temperatures and the symbol T appears, you can be fairly sure that this will represent a temperature even if the author forgets to define it as such. However, this is not good practice and all symbols should normally be defined.

A few symbols also have specialized mathematical meanings such as the Greek letter delta (i.e. Δ or δ) which is used to denote a small change in a variable. If temperature in the lower crust increased, due to some thermal event, by a small amount (say 10°C) this temperature change is given the symbol ΔT (or δT). If the original temperature was T°C the increased temperature is then $(T + \Delta T)$°C. Another, well known, example of reserving a symbol for a particular mathematical purpose is the use of π to denote the number 3.14 159 . . . , again this is such common usage that you will virtually never see π defined in a book or a paper. In this case, however, this is acceptable since this convention is universally adopted throughout mathematics and the sciences. Other examples of specialized mathematical meanings for symbols will be covered as we come across them in this book.

1.4 Subscripts and superscripts

Another feature of mathematical expressions, which some people find confusing, is the use of subscripts and superscripts. Subscripts are usually used to qualify the meaning of a symbol. For example, if T is used in an expression to denote temperature as a function of depth in the earth, then T_0 might well be used to denote the temperature at the Earth's surface (i.e. at depth = 0.0 metres). Similarly, if a sandstone and a shale are being compared, their densities might be given the symbols ρ_{sand} and ρ_{shale}, respectively, and their porosities would be given the symbols ϕ_{sand} and ϕ_{shale}. The use of subscripts is no different from the use of any other strange character to denote a quantity. Subscripts simply clarify the meaning of a particular symbol.

In sharp contrast, superscripts (often called **exponents**) have a definite mathematical meaning. A superscript is an instruction to raise a number to a power. Thus a^2 means 'square a', a^3 means 'find the cube of a' and a^n means 'multiply a by itself n times'. Whilst on the subject of raising numbers to a power, it is worth briefly reviewing a couple of simple points that will be needed later on. There are three manipulations in particular that you should be familiar with,

$$x^a x^b = x^{a+b} \tag{1.4}$$

$$x^a / x^b = x^{a-b} \tag{1.5}$$

and

$$(x^a)^b = x^{ab} \tag{1.6}$$

For example, one hundred (= 10^2) times one hundred equals ten thousand (= 10^4), i.e.

$$10^2 \times 10^2 = 10^{2+2} = 10^4$$

and the cube of four ($4 = 2^2$) is sixty-four ($64 = 2^6$), i.e.

$$(2^2)^3 = 2^{2\times 3} = 2^6$$

Question 1.2 Simplify and, where possible, evaluate the following expressions

(i) $5^{100}.5^4$;
(ii) $(5^{100})^4$;
(iii) $x^2.x^3$;
(iv) $Depth^2.Depth^3$;
(v) $(T_0^3)^4$ where $T_0 = 10$.

Number	Power of 10
1 000	10^3
10 000	10^4
100 000	10^5
1 000 000	10^6
1 billion	10^9

Table 1.3 Some large numbers expressed as positive powers of ten.

1.5 Very large numbers and very small numbers

Many quantities in geology are very large (e.g. the mass of the Earth) or very small (e.g. the mass of gold in one litre of sea water). It is therefore vital that you understand how to deal with very small or very large numbers. Two ways of talking about such extreme numbers are:

(i) The use of **scientific notation**;

(ii) The use of special, very large or very small, units.

Methods are also required for specifying very small fractions such as the fraction of a rare element contained in a mineral specimen.

Scientific notation (or standard notation) is the most flexible of the methods for discussing the very large and the very small. Table 1.3 shows how various large numbers can be represented by powers of ten. Note that in this book, and in most scientific literature, the American definition for one billion (one thousand million) is used rather than the British (one million million). The quick way to find out which power of ten to use for a particular number is simply to count the number of zeros. One million is 1 followed by 6 zeros and therefore equals 10^6.

This is fine for giving large numbers which are an exact power of ten. What about numbers such as two million? This is easy, two million is two times one million. This number is therefore written

$$
\begin{aligned}
2\ 000\ 000 &= 2 \times 1\ 000\ 000 \\
&= 2 \times 10^6
\end{aligned}
$$

This is an example of scientific notation for a large number. Other more complex numbers can also be dealt with such as 2 200 000. This is simply 2.2 times one million and can therefore be written as 2.2×10^6. The same number could equally well be thought of as 22 times one hundred thousand leading to 2 200 000 = 22×10^5. However, for scientific notation it is usual to have the multiplier falling between one and ten and so the former expression (i.e. 2.2×10^6) is preferred.

Small numbers can be dealt with in a similar manner. Table 1.4 shows how various small numbers are expressed as powers of ten. Again there is a quick

Question 1.3 Express the following numbers in scientific notation:
(i) 1000;
(ii) 2000;
(iii) 2500;
(iv) 2523;
(v) 23 000 000;
(vi) Seven billion.

Table 1.4 Some small numbers expressed as negative powers of ten.

Number	Power of 10
0.001	10^{-3}
0.0 001	10^{-4}
0.00 001	10^{-5}
0.000 001	10^{-6}
1 billionth	10^{-9}

way of determining the power of ten to use. Count the zeros including the zero before the decimal point. For example, 0.0 001 has four zeros in total and is written as 10^{-4}. Don't worry if it is not clear to you why a negative power of ten can be used to express these small numbers, this will be explained further in Chapter 2. For now it is only necessary that you accept that it works. Extending this system to numbers which are not exactly a power of ten is then achieved by introducing a multiplier. For example, 0.0 002 is twice 0.0 001 giving

$$0.0\,002 = 2 \times 0.0\,001 = 2 \times 10^{-4}$$

Another example is 0.0 000 054 which is written

$$0.0\,000\,054 = 5.4 \times 0.000\,001 = 5.4 \times 10^{-6}$$

Question 1.4 Express the following numbers in scientific notation:
(i) 0.001;
(ii) 0.002;
(iii) 0.0 025;
(iv) 0.002 523;
(v) 0.0 000 023;
(vi) Seven billionths.

Table 1.5 List of prefixes used in the SI system.

Multiple	Prefix	Symbol	Example
10^{-18}	Atto	a	Attometre (am)
10^{-15}	Femto	f	Femtometre (fm)
10^{-12}	Pico	p	Picometre (pm)
10^{-9}	Nano	n	Nanometre (nm)
10^{-6}	Micro	μ	Micrometre (μm)
10^{-3}	Milli	m	Millimetre (mm)
1	No prefix		Metre (m)
10^{3}	Kilo	k	Kilometre (km)
10^{6}	Mega	M	Megametre (Mm)
10^{9}	Giga	G	Gigametre (Gm)
10^{12}	Tera	T	Terametre (Tm)

SI (Système International) units are an alternative to the use of scientific notation. In this system a new unit is introduced for each thousand-fold increase or decrease in size. For example, the basic unit of distance is the metre (m). The next unit up from this is the kilometre (km) which is one thousand times bigger. One thousand kilometres is one million metres and this unit is denoted the megametre (Mm). Continuing on up the sequence we get one billion metres (the gigametre, denoted Gm) and one million million metres (the terametre, denoted Tm). Moving in the opposite direction, one thousandth of a metre is a millimetre (mm) and one millionth of a metre is a micrometre (μm). Finally, we get one thousand millionth of a metre which is called a nanometre (nm). Exactly the same set of prefixes is used for any other SI unit. Thus, the mass units, starting from the very small and increasing one thousand-fold for each step, are the nanogram (ng), the microgram (μg), the milligram (mg), the gram (g), the kilogram (kg), the megagram (Mg), the gigagram (Gg) and the teragram (Tg). These prefixes, and a few extra ones, are summarized in Table 1.5.

It is worth noting that a frequently encountered error in the use of this system is to use 'K' rather than 'k' in, for example, kilometre (i.e. this is written km not Km). A capital K is reserved for the **Kelvin** scale of temperature and thus 'Km' is an abbreviation of 'Kelvin metres' not 'kilometres'. Another point to note is that one very commonly encountered unit, the centimetre (cm), is not an SI unit and its use should normally be avoided.

Apart from common usages such as kilometre and kilogram, the SI method for discussing the very large or the very small is not widely employed in geology. One exception is the use of ky and My for thousands of years and millions of years, respectively (N.B. ka and Ma are also frequently used to denote thousands of years and millions of years). Strictly speaking, these are not SI units since the SI unit of time is the second. Nevertheless, ky, My, ka and

Ma are very commonly used in the earth sciences and therefore need to be understood.

Question 1.5 How long, in years, is 31.6 gigaseconds? (Hint: First work out how many seconds there are in a year of 365.24 days.) Using scientific notation, how many seconds is this?

When it comes to denoting very small fractions, the usual approach is a simple extension of the percentage system. In percentage notation, the figure '23%' means 23 parts in every hundred. Thus, if a rock specimen is 23% iron by weight, it contains 23 grams of iron in every 100 grams of rock. However, this is not a convenient system for discussing trace elements present in very small fractions of a per cent. Small proportions can be represented by talking about parts per million (ppm) or parts per billion (ppb). The rock specimen might contain 17 ppb of the element lanthanum. This means that every billion grams will contain 17 grams of lanthanum. It might also contain, say, 10 ppm of gold. Thus every million grams of rock will contain 10 grams of gold (i.e. 10 grams of gold in every metric tonne of rock).

Question 1.6 Express 0.01% in ppm.

1.6 Manipulation of numbers in scientific notation

Scientific notation is used frequently both in this book and throughout geological literature. You therefore have to know how to add, subtract, multiply and divide numbers expressed in this way.

The trick with addition or subtraction is to use the same power of ten for all numbers. For example, the net rate of increase in mountain height is given by the rate of uplift, which increases mountain height, minus the rate of erosion which tends to reduce mountain height. If the rate of uplift is 3×10^{-3} m/y whilst the rate of erosion is 5×10^{-4} m/y, the net rate of increase in the mountain height is

$$\text{Rate of increase in height} = \text{Rate of uplift} - \text{Rate of erosion}$$
$$= (3 \times 10^{-3}) - (5 \times 10^{-4}) \qquad (1.7)$$

The problem here is that the first number has an exponent of -3 whilst the second has an exponent of -4. However, the rate of erosion can be expressed with an exponent of -3 as follows

$$5 \times 10^{-4} = 0.5 \times 10^{-3}$$

Note that the 5 has been reduced by a factor of ten (to give 0.5) whilst the 10^{-4} has been increased by a factor of ten (to give 10^{-3}). Thus, the overall effect is to leave the value unchanged. Replacing the rate of erosion by this new expression gives

Rate of increase in height = $(3 \times 10^{-3}) - (0.5 \times 10^{-3})$

Once the numbers have been expressed using the same power of ten, the subtraction can be performed

Rate of increase in height = 2.5×10^{-3} m/y.

> **Question 1.7** Evaluate the following:
> (i) $(2.5 \times 10^{109}) + (1.5 \times 10^{109})$;
> (ii) $(2.5 \times 10^{109}) + (1.5 \times 10^{108})$;
> (iii) $(2.5 \times 10^{211}) - (1.5 \times 10^{211})$;
> (iv) $(2.5 \times 10^{211}) - (1.5 \times 10^{210})$.

Multiplication and division are more straightforward. The trick here is to rearrange the expressions so that all the multipliers are together and all the powers of ten are together. Some examples should illustrate this:

$$(2.5 \times 10^4) \times (3.0 \times 10^3) = (2.5 \times 3.0) \times (10^4 \times 10^3)$$

where, at this point, no calculation has been performed. The multipliers and powers of ten have simply been collected together. The calculations implied by the two bracketed terms can then be evaluated to give

$$(2.5 \times 3.0) \times (10^4 \times 10^3) = 7.5 \times 10^7$$

Similarly, for division

$$\begin{aligned}(5 \times 10^4)/(2.5 \times 10^3) &= (5/2.5) \times (10^4/10^3)\\ &= 2 \times 10^1 = 20\end{aligned}$$

Combined examples are also done this way, e.g.

$$\begin{aligned}\frac{(2.5 \times 10^4) \times (3.0 \times 10^3)}{7.5 \times 10^6} &= \frac{2.5 \times 3.0}{7.5} \times \frac{10^4 \times 10^3}{10^6}\\ &= (7.5/7.5) \times (10^7/10^6)\\ &= 1.0 \times 10^1\\ &= 10.0\end{aligned}$$

> **Question 1.8** Evaluate the following:
> (i) $(2 \times 10^{200}) \times (3 \times 10^{100})$;
> (ii) $(4 \times 10^{110})^2$;
> (iii) $(4 \times 10^{107})/(2 \times 10^{107})$;
> (iv) $(6 \times 10^{100})/(3 \times 10^{50})$.

Question 1.9 If the mass of the Earth is 5.95×10^{24} kg and the volume is 1.08×10^{21} m^3, calculate the average density. (Note that density is mass divided by volume).

I will finish this section with a few words of warning about using calculators for performing these sorts of calculations. There are two problems which this frequently causes. Firstly, many students will write down the results in a similar way to the manner in which they appear on the calculator display. Thus, if the correct answer is 3.01×10^8, this appears in the calculator something like [3.01 ⁸]. There is a strong temptation to write this down as 3.01^8 which means 3.01 to the power 8 rather than 3.01 times 10 to the power 8. The second problem is that many students would enter this number by typing the following buttons: [3], [.], [0], [1], [×], [1], [0], [exp], [8] which gives a display reading [3.01 ⁹]. This is because the correct sequence of buttons should have been: [3], [.], [0], [1], [exp], [8] since the [exp] button means multiply by 10 to the power of the following number. Try entering 3.01×10^8 in the two ways suggested above, and you should see what I mean. In general, I would strongly recommend that you perform calculations involving powers of 10 by using the methods shown in the earlier examples.

1.7 Use consistent units

Whenever a calculation is performed, all values used must be expressed using the same units. For example, in the calculation given above for finding the rate of rise of a mountain

$$\text{Rate of increase in height} = \text{Rate of uplift} - \text{Rate of erosion} \qquad (1.8)$$

the rate of uplift and the rate of erosion must be given using the same units. Thus, if the rate of uplift was given as

$$\text{Rate of uplift} = 3 \times 10^{-3} \text{ m/y}$$

whilst the rate of erosion was given as

$$\text{Rate of erosion} = 1 \text{ m/ky}$$

the calculation cannot be performed using these figures since the first figure has units of metres per year whilst the second has units of metres per thousand years. One of the two figures must be converted to the form of the other. In this case it is probably easiest to rewrite the rate of uplift as

$$\text{Rate of uplift} = 3 \text{ m/ky}$$

which is the same as 3×10^{-3} m/y since the amount of uplift in one thousand years is 1000 times more than that in one year (i.e. $10^3 \times 3 \times 10^{-3} = 3$).

Equation 1.6 can then be evaluated to give a net rate of mountain rise equal to 2 m/ky.

Similarly, in the lake sedimentation problem (Eqn. 1.1) all figures must have consistent units. Thus, if the age is quoted in years and the depth is quoted in metres, the sedimentation constant will have units of years/metre. If a depth is given in centimetres whilst k is given in years/metre, the depth must first be converted to metres before the calculation of age is done.

Question 1.10 Using Eqn. 1.1 and a sedimentation constant of 1000 years/metre, find the age of sediment buried at a depth of 30 cm.

1.8 Spreadsheets

Mathematics and computers are complimentary tools in quantitative science. Mathematics tells us what to calculate whilst computers are increasingly used to perform the final, usually numerical, calculations. Specialized software is often used to perform computations specific to a particular set of problems. However, some computer packages are much more general and can be used to solve many different problems. Of these more general purpose programs, **spreadsheets** must be the most widespread, useful and easy to use.

Spreadsheets consist of grids into which text, numbers or formulas can be typed. Figure 1.2 shows a very simple spreadsheet which lists the number of sites visited on four successive days of fieldwork. All **cells** contain precisely what you see in Fig. 1.2 (i.e. text or a number) apart from cell B7 which contains the formula = B2 + B3 + B4 + B5, i.e. an instruction to add together the contents of cells B2 to B5. Note that the same result could have been achieved if the formula = *sum* (B2 : B5) had been used instead.

The spreadsheet used to create Fig. 1.2 (it's called *Example.xls*) can be obtained using a **Web Browser** (e.g. Netscape or Internet Explorer) from www.gl.rhbnc.ac.uk (use the *links* button at the top of the page). Alternatively, you can access the same files using anonymous ftp for ftp.gl.rhbnc.ac.uk to log into the *pub* directory. This web site also contains many other spreadsheets (a complete list appears in *Index.xls*). The sheets are associated with various parts of this book and are designed to improve your understanding or to help you apply the mathematics you have learned to your own problems. These spreadsheets have been written using Excel version 7.0 but other packages (e.g. Lotus 123) should be able to read them provided they are reasonably recent versions. Most of these spreadsheets are password protected to prevent you from accidentally corrupting them but, if you are a confident spreadsheet user, you are welcome to alter them in any way you like. All sheets and workbooks are protected using the password *maths for*

	A	B
1	**Day**	**Number**
2	Monday	1
3	Tuesday	3
4	Wednesday	5
5	Thursday	2
6		
7	Total	11

Fig. 1.2 Example spreadsheet. All cells here contain either text or a number apart from cell B7 which contains a formula whose result is 11.

geologists. It is quite possible to use this book without using the spreadsheets but, if you do obtain them, I believe that you will find them helpful.

Many of you probably already know how to use spreadsheets but, even if you do, have a look at *Intro.xls* which will show you several spreadsheet features we will be using later. It will also help you revise what you have learned in Sections 1.2 and 1.6 above.

1.9 Further questions

1.11 If $\Omega_d = 3.1 \times 10^4$ and $\mu = 2.7 \times 10^{-2}$ evaluate $\Delta J = \mu / \Omega_d$

1.12 The Earth gains mass every day due to collision with (mostly very small) meteors. Estimate the increase in the Earth's mass since formation assuming that the rate of collision has been constant and that

$\Delta M = 6 \times 10^5$ kg/day
$A_e = 4.5 \times 10^9$ years

where ΔM is the present-day rate of mass gain and A_e is the age of the Earth. What is this mass gain as a fraction of the total present mass of the Earth, M_e, where

$M_e = 5.95 \times 10^{24}$ kg

Re-express this answer in ppb. Given that the Earth is believed to have formed by a process of accretion, has the rate been constant throughout the Earth's history?

1.13 Calculate the volume of the Earth using the expression

$$V = \frac{4\pi r^3}{3}$$

where r is the Earth's radius (equal to 6.37×10^6 m). Note that this method assumes that the Earth is a perfect sphere.

1.14 How long would it take to travel 100 km at 20 km per hour? The following problem is identical in form:
The North Atlantic Ocean is getting wider at an average rate, v_s, of around 4×10^{-2} m/yr and has a width, w, of approximately 5×10^6 m.
(i) Write an expression giving the age, A, of the North Atlantic in terms of v_s and w assuming the present-day spreading rate is typical of the ocean's entire history.
(ii) Evaluate your expression by substituting the values given above.

1.15 In simple models of mountain formation, the mountain is supported by thickened crust such that

$$\Delta z = h\ \rho_c / \Delta\rho$$

where Δz is the amount of crustal thickness, h is the mountain height, ρ_c is the density of the crust and $\Delta\rho$ is the density contrast between the crust and the underlying mantle. Calculate the increase in crustal thickness under a mountain of height 4×10^3 m if the crustal density is 2.5×10^3 kg/m^3 and the density contrast is 500 kg/m^3.