ES302 Class Notes – Trigonometric Applications in Geologic Problem Solving

(updated Spring 2016)

I. Introduction

a) Trigonometry – study of angles and triangles
   i) Intersecting Lines
      (1) Points of intersection

   ii) Polygons – multisided geometric features
      (1) Triangles, squares, hexagons, irregular-sided shapes
      (2) Multi-sided Polygons – any polygon can be subdivided into a set of mutually intersection triangles
   iii) Angles – described by two intersecting lines at a vertex or point of intersection

b) Representation of Geospatial Elements
   i) Lines – collection of a set of points
      (1) E.g. faults, dikes, joints

   ii) Polygons – closed set of intersecting lines
      (1) E.g. geologic map units

c) Geometry and Trigonometry form the basis of mapping and surveying
   i) Triangulation Surveys – map points and positions are located by measuring angles formed by the line of sight between the points

      (1) A network of points can be described and positioned in terms of a an interconnected network of triangles

         (a) The vertices of triangles form a series of map points

      (2) Measurement criteria

         (a) Distance between between points = lengths of the side of triangles
         (b) Angles between legs of triangle
            (i) Angles measured in terms of map bearings, relative to E-N-S-W

II. Angular Measurement

a. Angular Measurement (based on circle)

   i. Full Circle = 360 degrees
      1. 1 degree = 1/360 th of circle

         (1) Subdivisions of Degree
            (a) 1 degree = 60 minutes
            (b) 1 minute = 60 seconds
            (c) 1 degree = 60 min x 60 sec/min = 3600 sec

         (2) Famous Angular Measurements
            (a) Right Angle = 90 degrees

(b) (Straight Angle) Line = 180 degrees
(c) Circle = 360 degrees
(d) Acute Angle < 90 degrees
(e) Obtuse Angle: between 90-180 degrees
(f) Complementary Angles – two angles add up to 90 degrees

2. Radians – unit of angular measurement based on the length of an arc circumscribed by a circle

   a. Circumference of Circle = $2\pi r$,
      where π = circumference of circle / diameter = 3.14, and r = radius of circle

   b. Circle = 360 degrees = 2π radians; 180 degrees = π radians

3. Circle Geometry
   (a) Circumference = $2\pi r$
   (b) Diameter = $2r$
   (c) Area = πr^2

   where r = radius, c = circumference, $\pi = c/d = c/2r$

III. Trigonometric Functions

a. Triangle Geometry

   i. 3 angles (A, B, C)
   ii. 3 opposite sides (a, b, c)

b. Triangles
   i. Right Triangles – one angle = 90 degrees
   ii. Non Right Triangles – no angles = 90 degrees
   iii. Law of triangles = all interior angles add up to 180 degrees

c. Right Triangles and Trig Functions

   i. Hypotenuse – side opposite right angle
   ii. Sin = opposite / hypotenuse (ratio of length of opposite side / length of hypot)
   iii. Cos = adjacent / hypotenuse (ratio of length of adjacent side / length of hypot)
   iv. Tan = opposite / adjacent (ratio of length of opposite side / length of adjacent)
   v. Pythagoras' Theorem – square of length of hypotenuse = sum of squares of the other two sides of the triangle

$$a^2 + b^2 = c^2$$

d. Angles and Sides of Any Polygon (new spring 2016)

No. Sides = n
Sum Interior Angles = $(n-2)*180^o$
Each Equilateral Angle = $(n-2)*(180^o/n)$

Triangle:	n = 3	Sum Interior Angles = (3-2)*180 = 180	Each Angle = $(3-2)*(180/3)= 60^o$
Rectangle	n = 4	Sum Interior Angles = (4-2)*180 = 360	Each Angle = $(4-2)*(180/4)= 90^o$
Hexagon	n = 6	Sum Interior Angles = (6-2)*180 = 720	Each Angle = $(6-2)*(180/6)= 120^o$

e. Inverse Trigonometric Functions

i. If the sin, cos, or tan is known (i.e. the ratios of lengths of sides), then the angle can be determined by taking the inverse of the sin, cos, or tan

ii. Nomenclature
1. inverse sin = arc sin = $\sin^{-1}$
   a. e.g. $\sin^{-1}(0.602) = 37^o$
2. inverse cos = arc cos = $\cos^{-1}$
3. inverse tan = arc tan = $\tan^{-1}$

f. Raising Trig Functions to Other Powers

i. $Sin^3\theta = \sin\theta*\sin\theta*\sin\theta$

IV. Determining Unknown Angles and Distances with Triangles

a. General Rules for Triangles
i. 180 degree rule: sum of all interior angles A,B, C = 180 degrees

ii. Sin Rule

a/Sin(A) = b/Sin(B) = c/Sin(C)

where A,B,and C are interior angles; a, b, c are lengths opposite those angles

iii. Cosine Rule

$a^2 = b^2+c^2-2bc*\cos(A)$

$b^2 = a^2+c^2-2ac*\cos(B)$

$c^2=a^2+b^2-2ab*\cos(C)$

where A,B,and C are interior angles; a, b, c are lengths opposite those angles

iv. Additional Derivations of Cosine Rule and 180 rule

$Cos(A) = (b^2+c^2-a^2) / 2bc$ where angle A = inverse cos(A)

$Cos (B) = (a^2 + c^2 – b^2) / 2ac$ where angle B = inverse cos(B)

$C = 180 – (A + B)$ where C = angle C

In addition; A,B,and C are interior angles; a, b, c are lengths opposite those angles

V. Compass Bearings

4. Bearing- the direction from one point to another,

   a. Quadrant Method - angular degrees west or east or true north

      (1) e.g. N. 45 E or S. 65 W.

   b. Azimuth Method - angular degrees from 0-360, where north is 0^o or 360^o, east is 90^o, south is 180^o, and west is 270^o.

Examples of Bearings: Quadrant Method: N. 45 E Azimuth = 45
Quadrant Method: S. 20 W. Azimuth = 200

5. Measuring Bearings- To determine the bearing/direction from one point to another do the following:

   a. Draw a line from the first point to the second point
   b. using a protractor, center the origin of the protractor on the first point, align the bottom edge with the bottom edge of the map, and the 90 degree tic parallel with true north at the top of the map.
   c. Read the angle between north at the 90 degree mark of the protractor, and the line between the two points in question.
      (1) Give answer in either the quadrant or azimuth notation.

VI. Orientation of plane in space

i. Attitude: orientation of structural element in space

   1. planes: strike and dip
   2. lines: trend and plunge

ii. Bearing: compass direction orienting line in space relative to horizontal plane
   1. Azimuth vs. Quadrant

iii. Strike: compass bearing of horizontal line on a plane, formed by line of intersection between horizontal plane and inclined plane.

   1. infinite no. of parallel strike lines for any inclined plane

2. data representation: azimuth direction

iv. Dip: vertical angle between inclined plane and horizontal plane measured perpendicular to strike direction
    1. data: angle + direction of down dip orientation
        a. (i.e. direction water would run down the plane)

v. Apparent Dip: vertical angle between an inclined plane and a horizontal plane that is NOT measured perpendicular to strike direction

    1. Apparent dip will always be < than true dip

vi. Relevant Trig Functions to Dip and True Dip

$$\theta' = \tan^{-1}[\tan(\theta) * \cos(\alpha)]$$

where θ' = apparent dip, θ = true dip, α = angle between strike azimuth and exposure azimuth

and

$$\theta = \tan^{-1}[\tan(\theta')/\cos(\alpha)]$$

where θ' = apparent dip, θ = true dip, α = angle between strike azimuth and exposure azimuth