G302 - Basics Review of Math and Algebra

I. MATHEMATICS REVIEW

A. Decimal Fractions, basics and definitions

1. Decimal Fractions - a fraction whose deonominator is 10 or some multiple of 10 such as 100, 1000, 10000, etc.

8/10 = 0.8   79/100 = 0.79   183/1000 = 0.83

5925/10000 = 0.5925

1st place to right of decimal = tenths
2nd place to right of decimal = hundredths
3rd place to right of decimal = thousandths
4th place to right of decimal = 10 thousandths
5th place to right of decimal = 100 thousandths
6th place to right of decimal = millionths

Number	Powers of 10	Exponential Form
1,000,000	= 10 x 10 x 10 x 10 x 10 x 10	= 10^6
100,000	= 10 x 10 x 10 x 10 x 10	= 10^5
10,000	= 10 x 10 x 10 x 10	= 10^4
1000	= 10 x 10 x 10	= 10^3
100	= 10 x 10	= 10^2
10	= 10	= 10^1
1	= 1	= 10^0
0.1	= 1/10	= 10^{-1}
0.01	= 1/10 x 1/10	= 10^{-2}
0.001	= 1/10 x 1/10 x 1/10	= 10^{-3}
0.0001	= 1/10 x 1/10 x 1/10 x 1/10	= 10^{-4}
0.00001	= 1/10 x 1/10 x 1/10 x 1/10 x 1/10	= 10^{-5}

B. THE METRIC SYSTEM AND CONVERSION

1. Metric system- developed in Europe (France) in 1700's, offered as an alternative to the British or English system of measurement.

2. S.I./metric system involves measurements of length (meter), mass or weight (kilogram), temperature (celsius), time (second), and volume (litre).

3. Metric system based on powers of 10 and a decimal approach with prefixes attached to the basic units of measurment to indicate the power of 10 in question.

Greek prefixes > 1 base unit, Latin prefixes < 1 base unit

Peta = 10^{15}	P	
Tera = 10^{12}	T	
Giga = 10^{9}	G	
Mega = 10^{6}	M	e.g. 1 megameter = $1x10^6$ meter
Kilo = 10^{3}	k	1 kilometer = $1x10^3$ meters
Hecto = 10^{2}	h	1 Hectometer = $1x10^2$ meters
Deka = 10^{1}	da	1 Dekameter = $1x10^1$ meters
Base unit = 10^{0}		1 meter = 1 x 10^0 meters
Deci = 10^{-1}	d	
7. Centi = 10^{-2}	c	and so on
8. Milli = 10^{-3}	m	
9. Micro = 10^{-6}	μ	
10. Nanno = 10^{-9}	n	
11. Pica = 10^{-12}	p	

In-Class Problem: A mini-van sells for 33,220 dollars, express it's prices in kilodollars and Megadollars.

The movement of the decimal point to the left or right of the given quantity of a unit is all that is needed to change a given type of unit to the next higher or lower unit:

e.g. 1 m = 10 dm = 100 cm = 1000 mm = 1,000,000 um
1 m = 0.1 Dam = 0.01 Hm = 0.001 Km = 0.0000001 Mm

4. METRIC MEASUREMENT OF DISTANCE

a. Based on the meter (analogous to the yard in English system)
1 Km = 1000 m, 1 Hm = 100 m, 1 Dam = 10 m, 1 m = 1 m,
1 dm = 0.1 m, 1 cm = 0.01 m, 1 mm = 0.001 m, 1 um = 0.000001 m

b. Conversion of One metric unit to another

e.g. convert 8.9 km to m: 8.9 km $\frac{1000 \text{ m}}{1 \text{ km}}$ = 8900 m

e.g. convert 1230 m to km: 1230 m $\frac{1 \text{ km}}{1000 \text{ m}}$ = 1.23 km

5. METRICATION OF AREA (length x length)

a. SI units: km^2, m^2, cm^2, etc.

b. Metric equivalent of Acre = Hectare (Ha) = 100 m x 100 m which equals 10,000 m^2; i.e. 10,000 m^2/Ha

e.g. determine the no. of hectares in a plot of land: 1.6 km x 1.2 km = 1600 m x 1200 m = 1,920,000 m^2 (1 Ha/10,000m^2)= 192 Ha

6. METRICATION OF VOLUME (length x length x length)

a. volume- the amount of space within a container or enclosed within a solid

b. SI units of volume: cubic meters which can be equated to litres.

c. Can use same metric-prefix approach as given for meters, can be used with litres as well

e.g. 1 l = 1000 ml = .001 kl and so on
e.g. convert 17 litres to milliters:
17 l (1000 ml/l) = 17,000 ml

d. E.g. of problems converting volume in metric system

(1) Find the volume in liters of a rectangular tank (lxwxh) 2 m x 20 dm x 28 cm

7. METRICATION OF MASS
a. Mass - quantity of material contained in a given body
(1) Weight - measure of the force of gravity upon a given body.

Thus mass and weight are interchangeable under a given force of gravity, but may differ in cases of 2 different gravitational forces (e.g. a given mass will have different weights on the earth as compared to the moon (G moon = 1/6 G earth), but the mass or quantity of material occupying space will be same on earth as on the moon).

b. Metric unit of measuring mass = gram, kilogram, etc.
(1) converting from volume to capacity to weight:

1000 cu. cm = 1000 ml = 1000 gram of pure water
For pure water: 1 L = 1 Kg, thus 1 gm of water = 1 ml of water = 1 cu. cm

c. E.g. of metric conversions: convert 2700 mg to grams
2700 mg (1 gm/1000 mg) = 2.7 grams

8. METRIC MEASUREMENT OF TEMPERATURE
a. Metric unit = celsius, English unit = Farenheit
b. water freezes at 32^o F = 0^o C water boils at 212^o F = 100^o C
c. Conversion Factors:

(1) From C to F: $F = 9/5C + 32^o$
(2) From F to C: $C = 5/9(F - 32^o)$

(a) E.g. convert 40 C to F
$F = 9/5(40) + 32 = 104^o$ F

d. CONVERSION FROM ENGLISH SYSTEM TO METRIC AND VICE VERSA
(1) Conversion charts/factors given for units of length, area, volume, and weight/mass on p. 300.
(2)
(a) E.g. of conversion problems:
(b)

In-Class Exercise

Given that 1 yard = 0.9144 m, how many meters are there in 5360 yards?

C. Dimensional Analysis

1. Dimension – physical quantity, common physical quantities

L = distance
M = mass
T = time
L^2 = Area
L^3 = Volume
L/T = Velocity
L/T^2 = Acceleration
ML/T^2 = Force
ML^2/T^2 = Energy

2. Dimensionally-consistent formulas; units and dimensions must be consistent and algebraically sound; also refered to as “dimensionally homogenous” values

(a) *Dimensional Analysis* – process of algebraically balancing dimensions and units in an equation.

Example Equation: $x = x_o + vt$, where x = distance, v = velocity, t = time.

Dimensional Analysis of example: L = L + (L/T)T = L + L thus, length = length

(b) *Dimensionless quantities*: ratios where the units cancel or are identical
e.g. 4 m / 2 m = 2 a dimensionless ratio of lengths

In-Class Exercise: Show that $x = x_o + v_o t + 0.5\ at^2$
where x and x_o = distances, t = time, v_o = velocity, and a = acceleration; use dimensional analysis to demonstrate that the equation is properly constructed.

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c. Dimensionally inhomogenous empirical equations

(1) equations that are derived from direct observation or experimentation, that do not possess dimensionally consistent units.

E.g. Manning's Equation derived from flow through an open channel (relationship derived from direct experimentation, data collection, and analysis of relationships):

$$V = (D^{2/3} * S^{1/2}) / n$$

where V = velocity (m/sec), D = channel depth (m) S = slope (dimensionless), and n = roughness factor (dimensionless)

D. Significant Figures

1. defined – the number of significant digits in a quantity is equal to the number of digits that are known with certainty

   e.g. A ball is rolls 21.2 cm in 8.5 sec, the velocity = L/t = 21.2 cm/8.5 sec = 2.491176 cm / sec

   however, the minimum no. of siginicant digits in the calculation is 2 for the value of 8.5 sec, thus the answer must be limited to 2 significant digits, i.e. V = 2.5 cm/sec

E. Scientific Notation – using numbers in combination with powers of ten; standard notation:

e.g. $2500 = 2.5 \times 10^{3}$ e.g. $0.000036 = 3.6 \times 10^{-5}$

F. Orders of Magnitude

1. Magnitude refers to factors of powers of 10

   e.g. in comparing 100 to 1000, 1000 is one order of magnitude greater than 100

In-Class Exercise:

a. How many orders of magnitude are these two numbers apart: 100 vs. 10000000

b. Approximately how many orders of magnitude are these two numbers apart:

   3475 vs. 75849300

II.Algebra Review / Graph Function Review

A. Unit Conversion and Unit Management

1. Keeping track of unit dimensions in equations is very important
2. Unit algebra is based on simple unit cancelling

E.g. Given the fractional equation: $4 * \frac{2}{4}$ (note here "*" = times)

since there is a 4 in the numerator and 4 in the denominator, we can short-cut by simply cancelling out the 4 above, and 4 below (4/4 = 1)... and we find that the equation is equal to 2.

By analogy, given the algebraic equation: $Y * \frac{2}{Y}$ (note here "*" = times)

since there is a "Y" in the numerator and Y in the denominator, we can short-cut by simply cancelling out the Y above, and Y below (Y/Y = 1).... and we find that the equation is equal to 2.

By analogy, given that 1 mile = 5280 ft, we can convert 20,000 ft to miles by using unit algebra:

1) set up the equation so that the units you are trying to cancel are in the numerator and denominator

2) check to see if the end unit is the one you're looking for....

$20{,}000 \text{ ft} * \frac{1 \text{ mile}}{5280 \text{ ft}} = 3.79 \text{ miles}$ ... in this case the ft / ft cancels, leaving miles as the unit

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In-Class Exercise: given that 1 in = 2.54 cm, 1 ft = 12 in, and 1 mi = 5280 ft; How many centimeters are in 863 ft? Remember you are going from ft to cm, manage your units so that all cancel, except cm! Show all unit algebra.

B. Algebraic Manipulation of Exponents

1. Negative Exponents
   $a^{-n} = 1/a^n$
2. The zero power (any no. raised to the zero power = 1)
   $a^0 = 1$
3. Power of one (any no. raised to the 1st power = that number)
   $a^1 = a$
4. Multiplication (exponential nos. with the same base)
   $a^m * a^n = a^{m+n}$
5. Division
   $a^m/a^n = a^{m-n}$
6. Distribution
   $(a*b)^n = a^n*b^n$
   $(a^m)^n = a^{m*n}$

C. Dividing Fractions

1. When dividing by a fraction, invert the fraction and multiply

   e.g. 1/(1/4) = 1 * (4/1) = 4

   e.g. (m/sec)/sec = (m/sec)*(1/sec) = m/sec^2

D. Graphing Review

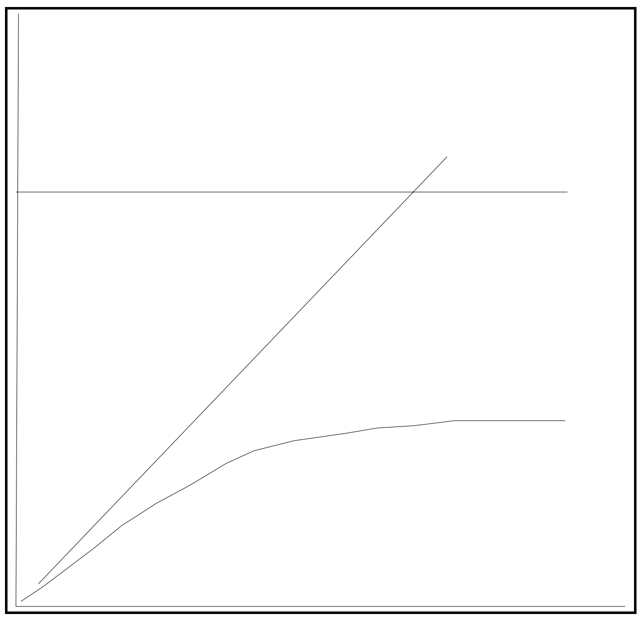
1. Axis
   a. Y axis = vertical axis (ordinate)
   b. X axis = horizontal axis (abscissa)

2. Graph Trends (see attached figures)
   a. Linear Increase / Decrease
   b. Constant
   c. Parabolic (curvilinear) Increase / Decrease

Linear Increase

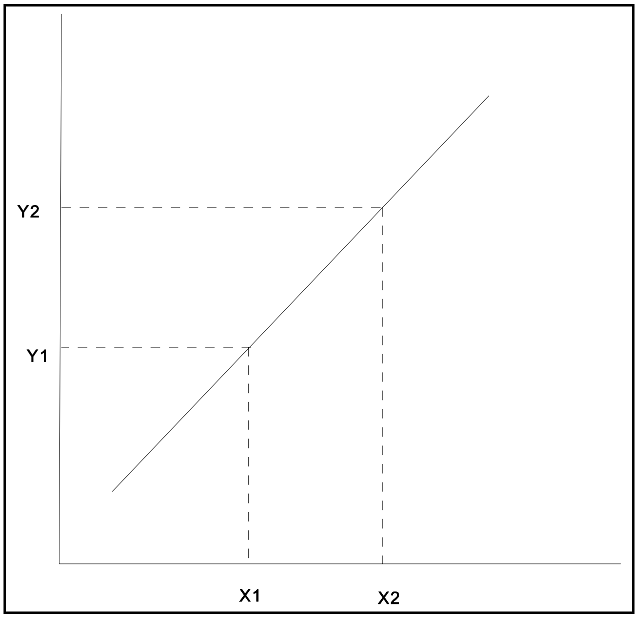
Constant

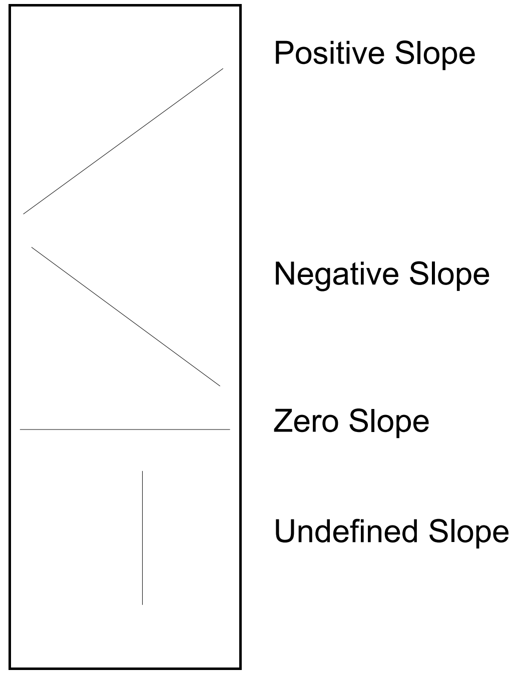
Parabolic



3. Determining Slopes of Lines

   a. slope of any line on a graph = rise / run = $(Y_2 - Y_1) / (X_2 - X_1)$





E. Rearranging equations algebraically

1. By using simple algebra, equations can be re-arranged to solve for other unknowns:

2. Examples

Given velocity and time, how to figure distance traveled during the time period?

Velocity $V = d/t$ rearranged to... multiply both sides of equation by t.... $d = V*t$

Given velocity and distance, how to figure time of travel?

Velocity $V=d/t$ rearranged to... $t = d/V$

Given acceleration and time, how to figure velocity acquired during the time period?

Acceleration $A = V/t$ rearranged to... multiply both sides of equation by t.... $V = A*t$

Example: you are driving a constant 50 km / hr for 35 minutes, how far have you traveled?

Example: you are accelerating in your car at 10 km/sec/sec for 90 sec, what is your velocity?

Common Conversion Factors

Time

1 b.y. = 1,000,000,000 years
1 m.y. = 1,000,000 years
1 year = 365 days
1 day = 24 hours
1 hour = 60 minutes
1 minute = 60 seconds

Length

1 mile = 5280 feet
1 foot = 12 inches
1 yard = 3 feet
1 inch = 2.54 cm
1 meter = 3.28 feet
1 meter = 100 cm
1 meter = 1000 mm
1 km = 1000 m
1 mile = 1.61 km
1 km = 0.62 miles

Mass / Weight

1 pound = 16 ounces
1 ton = 2000 pounds
1 kg = 1000 gm
1 ounce = 28 gm
1 kg = 2.2 pounds

Volume

1 gallon = 4 quarts
1 quart = 0.95 litres
1 litre = 1.05 quartz
1 litre = 1000 ml
1 ml = 1 cubic cm (of pure water)

In-Class Problem:
A city landscape covers approximately 10^8 m^2 in area. A rain storm drops 10 mm of rain over the area in a 12 hour period. Given that the average diameter of a raindrop is 4 mm, and assuming that a raindrop is spherical in shape, and that the volume of a sphere = $4\pi r^3/3$ (where r = radius of a sphere and π = 3.14); calculate how many rain drops fell on the city during the 12 hour period. Show all of your math work and unit algebra. (hint: calculate the total volume of rainfall on the city first)
