Appendix B: answers to problems

Chapter 1

1.1 *Age* = 1500 × *Depth*; *Depth* = 1 m, *Age* = 1500 years
Depth = 2 m, *Age* = 3000 years
Depth = 5.3 m, *Age* = 7950 years
Age = 3000 × *Depth*; *Depth* = 1 m, *Age* = 3000 years
Depth = 2 m, *Age* = 6000 years
Depth = 5.3 m, *Age* = 15 900 years

1.2 (i) 5^{104}; (ii) 5^{400}; (iii) x^5; (iv) $Depth^5$; (v) $T_0^{12} = 10^{12}$
1.3 (i) 1×10^3; (ii) 2×10^3; (iii) 2.5×10^3; (iv) 2.523×10^3; (v) 2.3×10^7; (vi) 7×10^9
1.4 (i) 1×10^{-3}; (ii) 2×10^{-3}; (iii) 2.5×10^{-3}; (iv) 2.523×10^{-3}; (v) 2.3×10^{-6}; (vi) 7×10^{-9}
1.5 1000 years = 3.16×10^{10} s
1.6 0.01% = 0.01 parts per hundred = 0.1 parts per thousand = 100 ppm
1.7 (i) 4×10^{109}; (ii) 2.65×10^{109}; (iii) 1×10^{211}; (iv) 2.35×10^{211}
1.8 (i) 6×10^{300}; (ii) 1.6×10^{221}; (iii) 2; (iv) 2×10^{50}
1.9 5509 kg m^{-3}
1.10 300 years
1.11 8.71×10^{-7}
1.12 Total mass gained = $\Delta M \times A_e = 6 \times 10^5 \times 365.24 \times 4.5 \times 10^9$
$= 9.86 \times 10^{17}$ kg
Fractional gain = $9.86 \times 10^{17}/5.95 \times 10^{24} = 1.66 \times 10^{-7}$ = 166 ppb
1.13 1.08×10^{21} m^3
1.14 (i) $A = w/v_s$; (ii) 1.25×10^8 years = 125 million years
1.15 20 km

Chapter 2

2.1 1.025 My
2.2 Gradient = (1 050 000 − 1 010 000)/(100 − 20) = 40 000/80
= 500 y m^{-1}
2.3 Rate = 3800 y m^{-1}, lake dried out 545 000 years ago. N.B. Your gradient should lie within about 500 y m^{-1} of mine and your intercept within about 10 000 years of mine

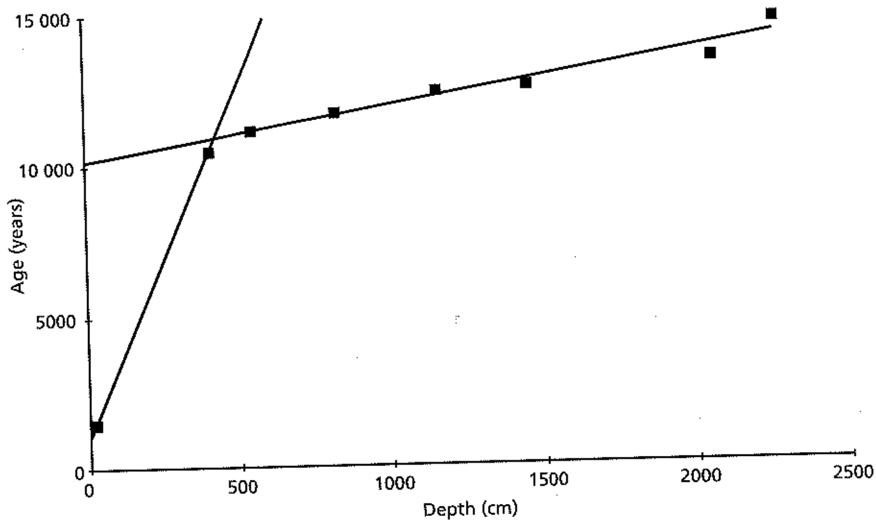

Fig. b1

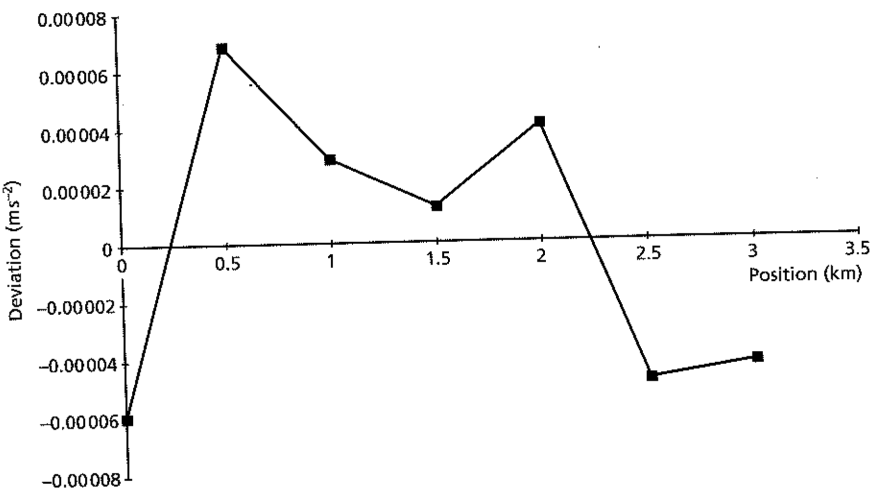

Fig. b2

2.4 $\tau = mP + \tau_0$. The graph should be a straight line of gradient m and intercept τ_0

2.5 $a = 2$, $b = -10$ and $c = 6$

2.6 Equation 2.6 gives 3403°C; Equation 2.8 gives 3648°C; Equation 2.8 is much closer to the true value of 3700°C

2.7 Your answers should be close to 0.45

2.8 $0.6 \times 2^{-2} = 0.6/2^2 = 0.6/4 = 0.15$

2.9 $\phi = 0.7\exp(-1/2) = 0.7 \times 0.6065 = 0.425$
2.10 25
2.11 See Fig. b1: (i) 23 y cm^{-1}; (ii) 1.8 y cm^{-1} (to within about 0.4 cm); (iii) 1030 years
2.12 $C = 200 \times 0.5^{5.5} = 4.42$ ppm
2.13 Graph should be straight line of gradient -10^{-7} and intercept 6.91. Age is 23 million years
2.14 See Fig. b2. Ore body lies between 0.5 km and 2 km
2.15 (i)

Z	p	Z	p
0	3	12	1.646 435
2	2.714 512	14	1.489 756
4	2.456 192	16	1.347 987
6	2.222 455	18	1.219 709
8	2.01 096	20	1.103 638
10	1.819 592		

(ii) p_0 is the accumulation rate in zero water depth. Z is the depth at which accumulation decreases to $p_0/e \sim 1$ m ky^{-1}

Chapter 3

3.1 k = Age / Depth; hence, $k = 3000/3 = 1000$ y m^{-1}
3.2 Age of top = Age − k. Depth = 60 000 − 5000 × 10 = 10 000 years
3.3 $w/x = [3y/(4z)]/[2y/(4z)] = 12yz/8yz = 12/8 = 1.5$
3.4 (i) $5(x + 2y) = 5x + 10y$; (ii) $5(x + 2.2y) = 5x + 11y$; (iii) $5.5(x + 2y) = 5.5x + 11y$; (iv) $5a(x + 2y) = 5ax + 10ay$; (v) $(x - 2y)(x + 2y) = x^2 - 4y^2$; (vi) $(x + 2y)^2 = x^2 + 4y^2 + 4xy$
3.5 Depth = $(1/k)$(Age − Age of top) = [$(1/k)$ Age] − [$(1/k)$ Age of top] = (Age/k) − (Age of top)/k
3.6 $6ax + 3ay = 3a(2x + y)$
3.7 See Fig. b3
3.8 $x = 1$ for both roots
3.9 2170 km and 10 550 km
3.10 $\rho = \rho_g[1 - (V_p/V)] = M/V$, hence $M = V\rho_g[1 - (V_p/V)]$
$= \rho_g(V - V_p)$

and so $\rho_g = M/(V - V_p)$
If $M = 205$ kg, $V = 0.11$ m^3 then average density = $M/V = 205/0.11$ $= 1864$ kg m^{-3}
For a porosity of 0.32, $V_p = 0.32 \times 0.11 = 0.0352$ m^3
From above $\rho_g = M/(V - V_p) = 205/(0.11 - 0.0352) = 2741$ kg m^{-3}

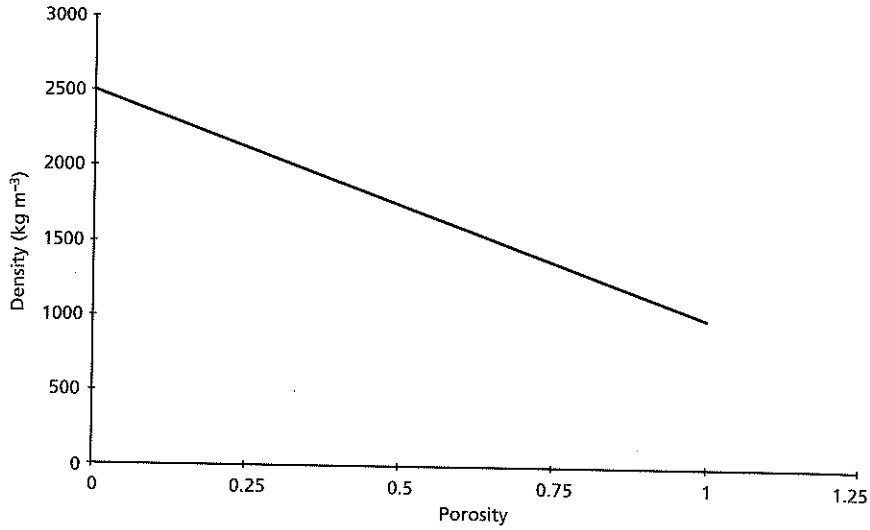

Fig. b3

3.11 $v_1 = \dfrac{2(\rho_p - \rho_f)gr_1^2}{9\eta}$ and $v_2 = \dfrac{2(\rho_p - \rho_f)gr_2^2}{9\eta}$, hence $\dfrac{v_1}{v_2} = \dfrac{r_1^2}{r_1^2} = \left(\dfrac{r_1}{r_2}\right)^2$

If $r_1 = 0.1$ mm and $r_2 = 1$ mm then $v_1 = 0.01v_2$. Hence, smaller particle settles at one-hundredth of the speed of the larger particle. Larger particle therefore takes 0.1 days = 2.4 hours to settle

3.12 (i) $b = -ax - (c/x)$; (ii) $b^2 = a^2x^2 + (c^2/x^2) + 2ac$, therefore $b^2 - 4ac = a^2x^2 + (c^2/x^2) - 2ac$;

(iii) Easiest way is $[ax - (c/x)]^2 = [ax - (c/x)][ax - (c/x)]$

$$= (a^2x^2 - ac) - (ac - c^2/x^2)$$
$$= a^2x^2 + (c^2/x^2) - 2ac;$$

(iv) $b^2 - 4ac = [ax - (c/x)]^2 = (b + 2ax)^2$ (from (i) above)

hence, $b + 2ax = \sqrt{b^2 - 4ac}$ giving $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ as required

Chapter 4

4.1 15 million years

4.2 11 ppb

4.3 $a = 1.1$; $b = 2$; $c = 3$

4.4 Approximation gives $3000 = (-1 \times 10^{-4})z^2 + z + 1000$

Further manipulation leads to $z = \dfrac{-1 \pm \sqrt{0.2}}{-2 \times 10^{-4}}$ approximating further that $\sqrt{0.2} = 0.5$ then gives $z = 2500$ km or $z = 7500$ km

4.5 $G = \dfrac{r^2 g}{M}$, therefore units$(G) = \dfrac{m^2\ m\ s^{-2}}{kg} = m^3\ kg^{-1}\ s^{-2}$

4.6 $T = 0a + T_0 = T_0$ as expected

4.7 (i) $\ln(t_0) = (x/X) + \ln(t)$

(ii) and (iii) $\ln(t_0) = (1/X) + \ln(5)$ and $\ln(t_0) = (4/X) + \ln(0.1)$

solving simultaneously gives $X = 0.767$ km and $t_0 = 18.4$ m

4.8 and 4.9 $a = -6.93 \times 10^{-13}$°C km^{-4}, $b = -5.55 \times 10^{-5}$°C km^{-2};

$c = 4390$°C

Predicted central temperature = 4390°C

Predicted surface temperature = 1010°C

4.10 (i) wrong; (ii) wrong; (iii) wrong; (iv) wrong

Chapter 5

5.1 (i) B = 60°; b = 10.1 cm; c = 11.5 cm

(ii) A = 25°; B = 135°; c = 2.4 cm

5.2 Church–transmitter distance = 3.2 km

Church–exposure distance = 4.5 km

Transmitter–exposure distance = 3.5 km

5.3 (i) 180° = π radians; (ii) 90° = $\pi/2$ radians; (iii) 270° = 1.5 π radians; (iv) 100° = $100\pi/180$ = 1.75 radians

5.4 See Fig. b4. From this $\sin(\theta) = \sqrt{3}/2$; $\cos(\theta) = 1/2$; $\tan(\theta) = \sqrt{3}$ For the other angle sine and cosine are swapped around and tangent is $1/\sqrt{3}$

5.5 142 m

5.6 22°

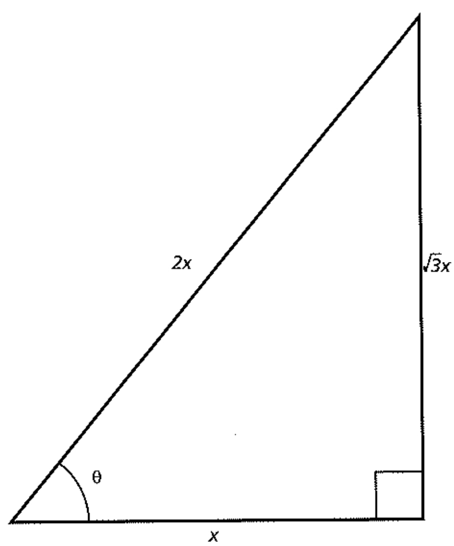

Fig. b4

5.7 (i) $C = 40°$; (ii) $B = 74.6°$; (iii) $a = 2.28$ km
5.8 $a^2 = b^2 + c^2 - 2bc\cos(90) = b^2 + c^2 - 0 = b^2 + c^2$
5.9 22.3° opposite side of length 200 m
108.2° opposite side of length 500 m
49.5° opposite side of length 400 m
5.10 (i) $B = 119.7°$, $C = 20.3°$, $a = 3.70$ km; (ii) $B = 74.6°$, $C = 65.4°$, $c = 2.82$ km; (iii) $C = 80°$, $b = 4.04$ km, $c = 4.60$ km
5.11 $\tan(\theta_C) = -4/-4 = 1$
$\tan(\theta_D) = -8/8 = -1$
5.12 48.2°
5.13 (i) Scalar; (ii) Vector; (iii) Scalar; (iv) Vector
5.14 See Fig. b5. N.B. Vector $\underline{e}$ has length of zero and is not shown

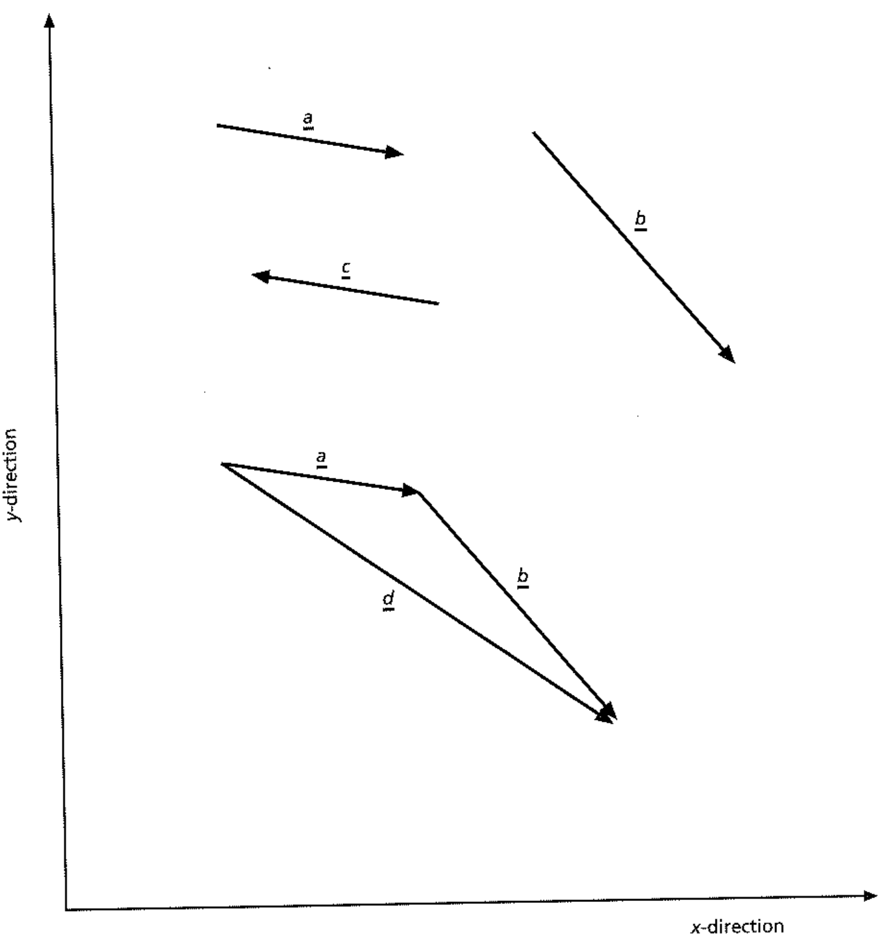

Fig. b5

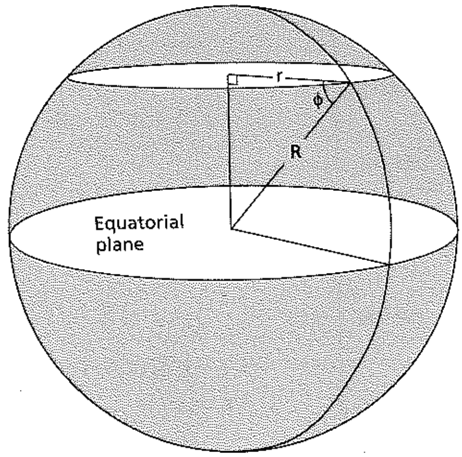

Fig. b6

5.15 $\cos(\theta) = x/a$, therefore $x = a\cos(\theta)$.$\sin(\theta) = y/a$, therefore $y = a\sin(\theta)$

5.16

Vector 1: x-component = 10 cos(60) = 5.0 m, z-component = 10 sin(60) = 8.66 m

Vector 2: x-component = 5 cos(65) = 2.11 m, z-component = 5 sin(65) = 4.53 m

Vector 3: x-component = 12 cos(45) = 8.49 m, z-component = 12 sin(45) = 8.49 m

Vector sum: x-component = 15.60 m, z-component = 21.68 m

Magnitude = 26.71 m. Dip = 54.3°

5.17 (i) cos(15°) = 0.966; (ii) sin(1.2 radians) = 0.932; (iii) $\tan^{-1}(0.5)$ = 26.6°; (iv) $\cos^2(27°)$ = 0.794; (v) $(\tan(0.5°))^{-1}$ = 115

5.18 See Fig. b6. From this $\cos(\phi) = r/R$ giving $r = R\cos(\phi)$

5.19 436 m

5.20 From section, tan(apparent dip) = 150/1000; from map, true direction of maximum dip = 162°; hence angle between section and dip section is 32°; equation 5.32 then gives true dip = 10.0°

5.21 352° E of N

5.22 11.3°

5.23 0.277 m s^{-1} at 57.2° E of N

Chapter 6

6.1 (i) See Fig. b7; (ii) See Fig. b8; (iii) See Fig. b9; gradient = −0.9, therefore $b = 0.9$

6.2 Mass = 55 000 kg; foot area = 2 m^2

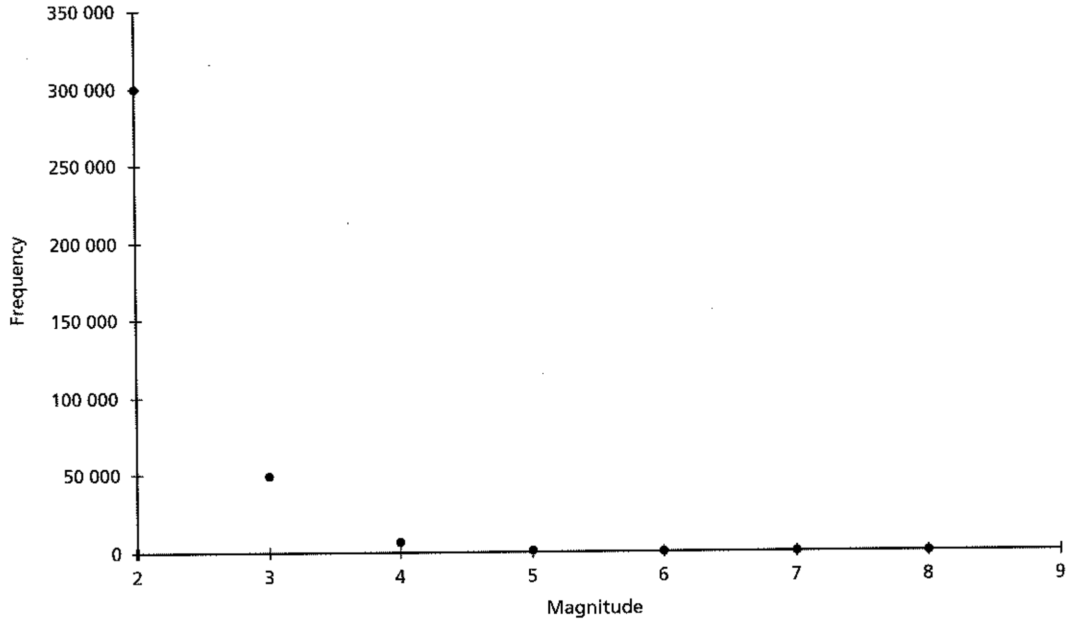
350 000
300 000
250 000
200 000
150 000
100 000
50 000
0
Frequency
2
3
4
5
6
7
8
9
Magnitude

Fig. b7

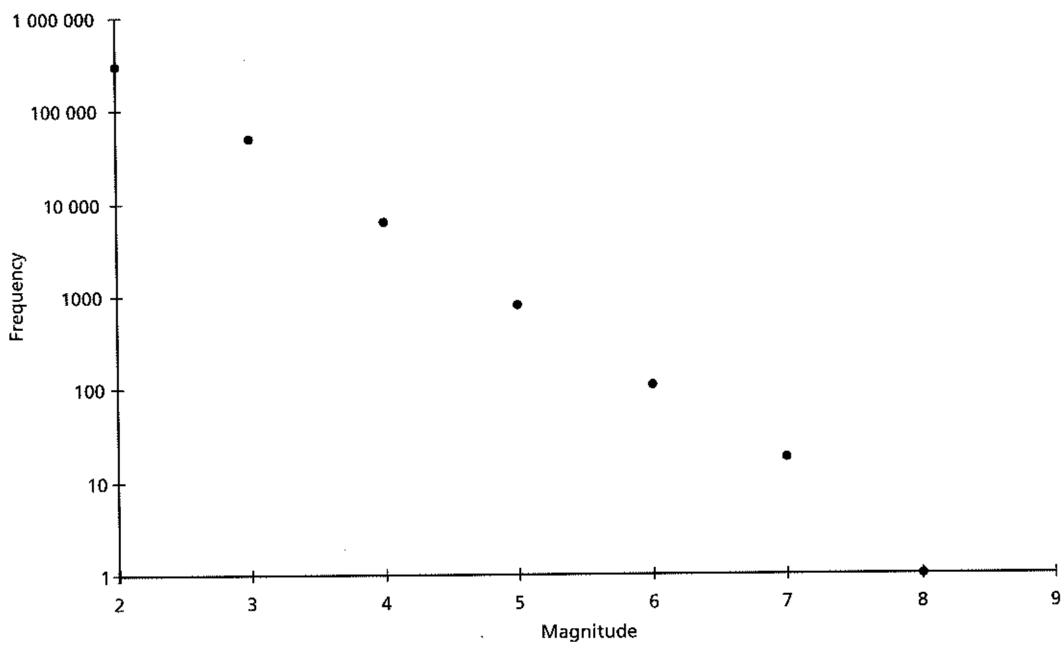
1 000 000
100 000
10 000
1000
100
10
1
Frequency
2
3
4
5
6
7
8
9
Magnitude

Fig. b8

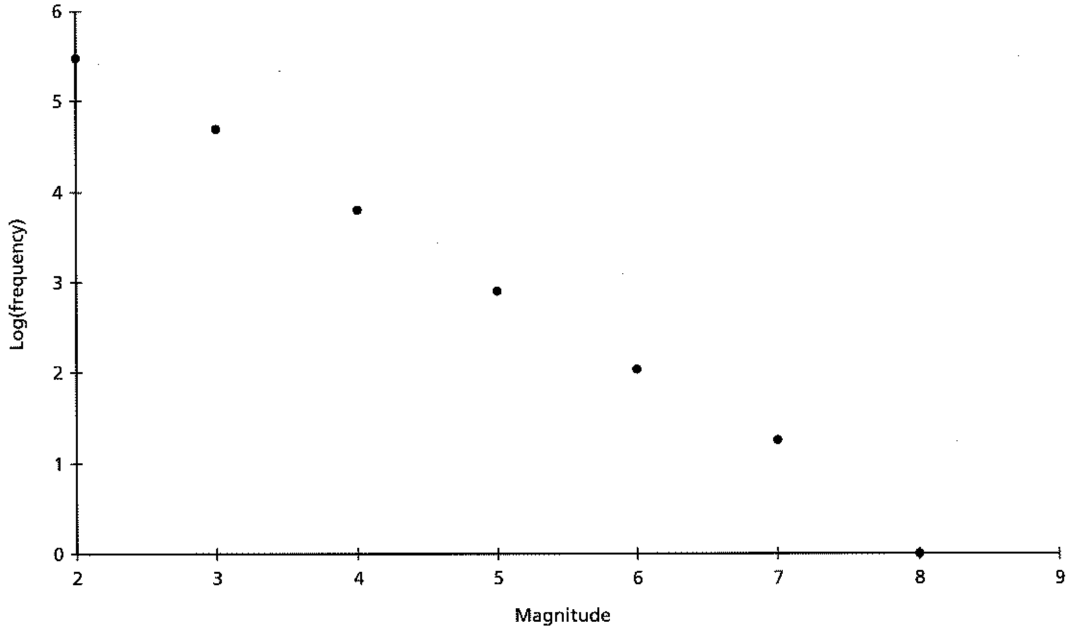

Fig. b9

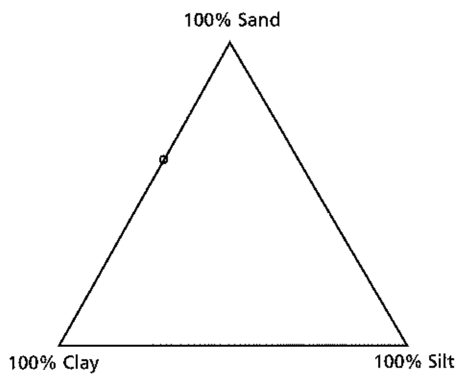

Fig. b10

6.3 See Fig. b10
6.4 See Fig. b11
6.5 10 mm in both cases
6.6 10°–20° = 12.50 mm; 70°–80° = 7.80 mm
6.7 10°–20° = 8.81 mm; 70°–80° = 11.01 mm
6.8 A triangular diagram
6.9 A simple x–y plot
6.10 A polar graph
6.11 (i) A great circle; (ii) Bed dip and pole dip differ by 90°. The dip directions differ by 180°; (iii) See table below and Fig. b12

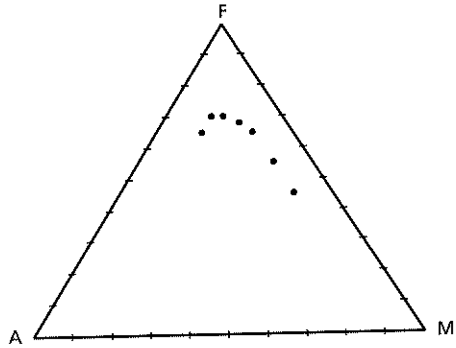

Fig. b11

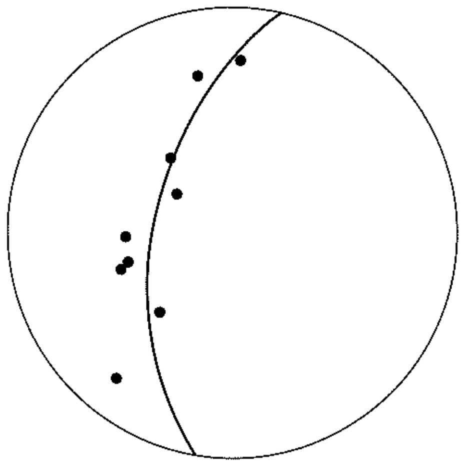

Fig. b12

Pole dip	Pole azimuth
20	362
25	350
65	311
50	250
54	270
20	215
50	220
53	253
53	326

(iv) Fig. b12 also shows a great circle which lies reasonably close to the data which is therefore consistent with a simple fold

6.12 (i) A polar plot of some kind, for example a stereonet; (ii) A triangular diagram; (iii) A log–normal graph with %TOC on the logarithmic axis

Chapter 7

7.1 TTTT, HTTT, THTT, TTHT, TTTH, HHTT, HTHT, HTTH, THHT, THTH, TTHH, THHH, HTHH, HHTH, HHHT, HHHH
Hence,

0 Heads	1 Head	2 Heads	3 Heads	4 Heads
1	4	6	4	1

Therefore 2 heads is the most likely result from 4 coin tosses
7.2 A sample
7.3 (i) mean = 337.6 g; (ii) median = 349 g; (iii) variance = 2838.24 g^2; (iv) population variance = 3153.6 g^2; (v) standard deviation = 56.16 g
7.4 $P_{2.1} = 0.964$; $P_{1.1} = 0.729$; $P_{1.0} = 0.683$; hence, $P_{1.06} = 0.711$. Thus, probability required = (0.964 – 0.711)/2 = 0.127 *Gauss.xls* gives 0.125
7.5 Gradient = 1974 y m^{-1}, intercept = 61.8 y
7.6 No, these ratios are indistinguishable within error
7.7 19 degrees of freedom gives t(95) = 2.1 as for the strike data. Hence, Dip(A) = 22.8 ± 1.2° and Dip(B) = 20.3 ± 1.8°
7.8 (i)

Range (°)	Frequency	Probability
0–30	9	0.45
30–60	0	0
60–90	2	0.1
90–120	2	0.1
120–150	0	0
150–180	0	0
180–210	3	0.15
210–240	0	0
240–270	1	0.05
270–300	2	0.1
300–330	0	0
330–360	1	0.05

(ii) See Fig. b13
(iii) Trend is towards NNE

7.9 (i) $s^2 = \dfrac{1}{N}\sum_{i=1}^{N}(w_i^2 + \bar{w}^2 - 2w_i\bar{w})$;

(ii) $s^2 = \dfrac{1}{N}\left(\sum_{i=1}^{N} w_i^2 + \sum_{i=1}^{N}\bar{w}^2 - \sum_{i=1}^{N} 2w_i\bar{w}\right)$;

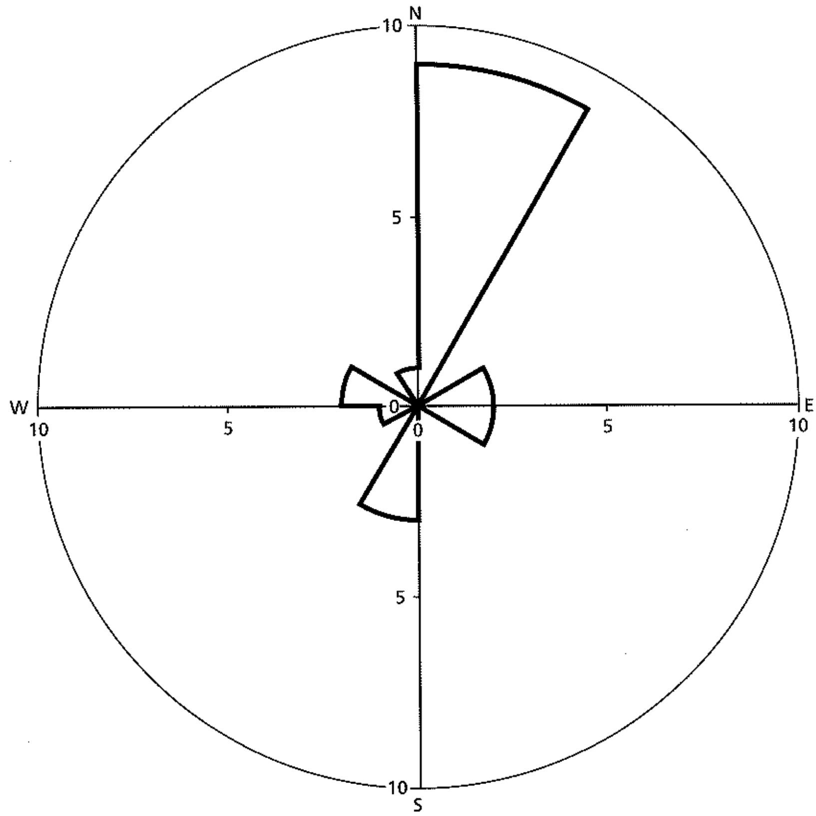

Fig. b13

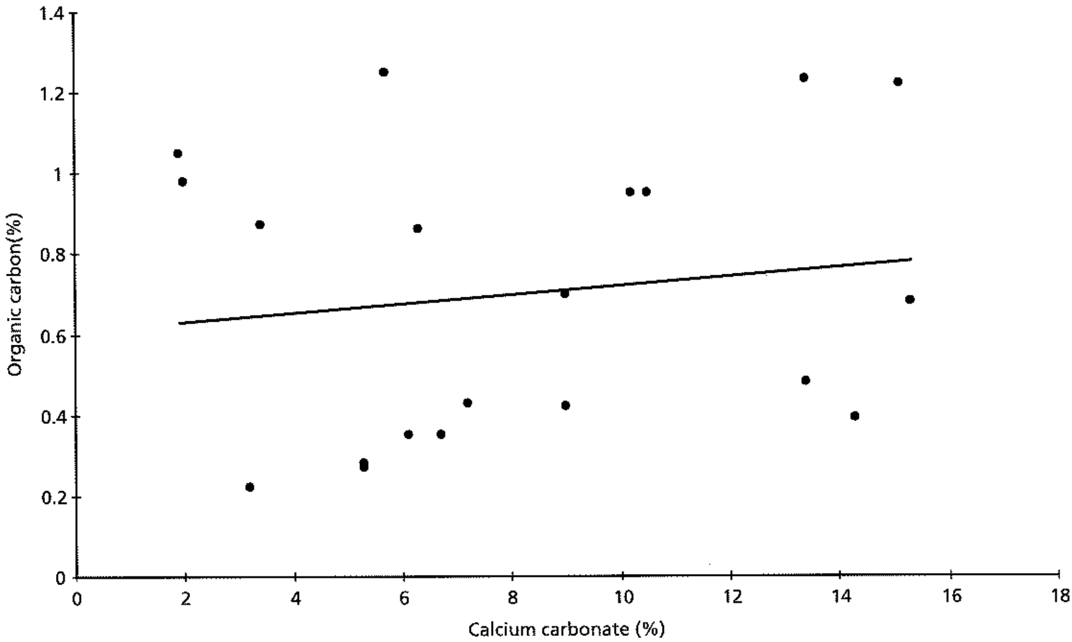

Fig. b14

(iii) $s^2 = \left(\frac{1}{N}\sum_{i=1}^{N} w_i^2\right) + \left(\frac{1}{N}\,\overline{w}^2 \sum_{i=1}^{N} 1\right) - \left(2\overline{w}\,\frac{1}{N}\sum_{i=1}^{N} w_i\right)$;

(iv) $s^2 = \overline{w^2} + \overline{w}^2 - 2\overline{w}^2 = \overline{w^2} - \overline{w}^2$

7.10 Skew = −0.898

7.11 (i) Organic C = 0.011 × calcium carbonate + 0.606; (ii) See Fig. b14; (iii) Regression is a poor fit to the data

7.12 Subsidence at Tona = 1.16 × subsidence at Puig d'Olena − 25.0

7.13

	Mount Monger	Emu
Mean	63.87	65.61
Number of values	9	10
Standard deviation	2.60	3.22
Standard error	0.87	1.02
95% confidence	2.00	2.31

Hence, mean SiO_2 content is 63.9 ± 2.0% at Mount Monger and 65.6 ± 2.3% at Emu. The large overlap implies that there is no significant difference in mean SiO_2 content in the two locations

Chapter 8

8.1 Gradient is about −30% per km

8.2 Gradients are 4 and 2000 at $x = 2$ and $x = 1000$, respectively

8.3 If $y = x^3$ (1)

then $y + \Delta y = (x + \Delta x)^3$

$= x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3$ (2)

subtracting (1) from (2) gives $\Delta y = 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3$

dividing by Δx gives $\Delta y/\Delta x = 3x^2 + 3x\Delta x + \Delta x^2$

as Δx tends to zero this becomes $dy/dx = 3x^2$

8.4 If $y = x^3$ then this is of the form $y = x^n$ with $n = 3$. Hence, gradient is $dy/dx = nx^{n-1} = 3x^2$

8.5 (i) $dy/dx = 20x^{19}$; (ii) $dw/dz = e^z$; (iii) $dw/dx = zx^{z-1}$

8.6 (i) $dy/dx = 10x^9 + 1/\cos^2(x)$; (ii) $dy/dx = 2x + 3x^2 + 4x^3$; (iii) $dw/dz = 75e^z$; (iv) $d\alpha/d\theta = 10\cos(\theta) - 13\sin(\theta)$

8.7 $dT/dz = 4az^3 + 3bz^2 + 2cz + d$

with $a = -1.12 \times 10^{-12}$, $b = 2.85 \times 10^{-8}$, $c = -0.000\,310$, $d = 1.64$, $z = 1000$ gives $dT/dz = 1.1$°/km

8.8 (i) $d\alpha/dx = x^2.e^x + 2x.e^x$; (ii) $dy/dw = 6w.\sin(w) + 3w^2\cos(w)$; (iii) $dz/dx = \cos(x) - x\sin(x) + 3x^2.\tan(x) + x^3/\cos^2(x)$; (iv) $dB/d\sigma = 12\,\sigma^3.\ln(\sigma) + 3\sigma^3 + 34\sigma$

8.9 $u = x^4$ gives $du/dx = 4x^3$
$v = \sin(x)$ gives $dv/dx = \cos(x)$

hence, $\dfrac{dy}{dx} = \dfrac{v(du/dx) - u(dv/dx)}{v^2} = (\sin(x).4x^3 - x^4.\cos(x))/\sin^2(x)$

8.10 $x = \ln(y^2)$ (1)

let $u = y^2$ (2)

then $x = \ln(u)$ (3)

and $du/dy = 2y$ (4)

equation (3) gives $dx/du = 1/u$ (5)

chain rule is $dx/dy = dx/du \,.\, du/dy = (1/u).(2y)$

$= (1/y^2).(2y) = 2/y$

8.11 The measurements differ by less than their uncertainties after altitude correction. Hence, differences are not significant

8.12 $dy/dx = 6x + 2$

$d^2y/dx^2 = 6$

$d^3y/dx^3 = 0$

$d^4y/dx^4 = 0$

8.13 (i) $S = S_{max}(1 - e^{-0/\tau}) = S_{max}(1 - 1) = 0$; (ii) $S = S_{max}(1 - e^{-\infty/\tau}) = S_{max}(1 - 0) = S_{max}$; (iii) $S = S_{max} - S_{max}e^{-t/\tau}$, therefore $dS/dt = 0 + (S_{max}/\tau)e^{-t/\tau}$. At $t = 0$ this gives $dS/dt = S_{max}/\tau$; (iv) Result from (iii) gives $dS/dt = 0$ at $t = \infty$; (v) Result from (iii) gives $dS/dt = 3/50 = 0.06$ km/My or 60 m/My

8.14 (i) At $x = X$ salinity equation gives $3s_0 = s_0\,\alpha X/(\alpha X - X) = s_0\,\alpha/(\alpha - 1)$ Hence $3 = \alpha/(\alpha - 1)$ which can be rearranged to give $\alpha = 1.5$; (ii) If $s_0 =$ 30 ppm and $x = X/2$, salinity equation becomes $s = 45X/(1.5X - X/2) = 45/(1.5 - 0.5) = 45$ ppm; (iii) Use quotient rule and then simplify; (iv) If $X = 10\,000$ m then at $x = X/2$, $ds/dx = s/(\alpha X - x) = 45/(1.5 \times 10\,000 - 5000) = 0.0045$ ppm/m. In other words, salinity changes by 4.5 ppm over 1 km. Hence, 1 km seaward of centre salinity is $45 - 4.5 = 40.5$ ppm. Similarly, 1 km shoreward of centre salinity is $45 + 4.5 = 49.5$ ppm

8.15 (i) $du/d\theta = -2\ \cos\theta/\sin^3\theta$ and $dv/d\theta = 2\ \sin\theta/\cos^3\theta$; (ii) $d^2w/d\theta^2 = \alpha et[(2\ \sin\theta/\cos^3\theta) + (2\ \cos\theta/\sin^3\theta)] = 2\alpha et(\sin^4\theta + \cos^4\theta)/\cos^3\theta.\sin^3\theta$; (iii) If $\cos\theta = \sin\theta = 1/\sqrt{2}$ then $\cos^4\theta = \sin^4\theta = 1/4$, $\cos\theta.\sin\theta = 1/2$ and therefore $\cos^3\theta.\sin^3\theta = 1/8$. Substitute these into the above expression to get a result.

8.16 (i) $dt/dx = (-t_0/X)\exp(-x/X)$;

(ii) $dt/dx = (-10/5)\exp(-3/5) = -1.098$ m/km

8.17 (i) If $a = k.z$ then $da/dz = k$; (ii) $\Delta a = (da/dz).\Delta a = k\Delta a$;

(iii) $\Delta a = 3000 \times 0.1 = 300$ m

8.18 $h = x.\tan(\alpha) = 100.\tan(20°) = 36.4$ m
$dh/d\alpha = x/\cos^2(\alpha)$ Hence height error, $\Delta h = (dh/da).\Delta\alpha = \Delta\alpha\, x/\cos^2(\alpha)$
Angle error, $\Delta\alpha = 2° \times \pi/180° = 0.035$ radians.
Therefore, $\Delta h = 0.035 \times 100/\cos^2(20°) = 3.96$ m
Hence, cliff height is 36.4 ± 4.0 m

Chapter 9

9.1 $\int 10e^{2\alpha}d\alpha = 5e^{2\alpha} + k$

9.2 (i) $\int \cos(x)\,dx = \sin(x) + k$; (ii) $\int \xi^{10}\,d\xi = (\xi^{11}/11) + k$

9.3 $\int_0^X x^2dx = \left[\frac{x^3}{3}\right]_0^X = \frac{X^3}{3} - \frac{0}{3} = \frac{X^3}{3}$

9.4 $\int_0^X x^n dx = \frac{X^{n+1}}{n+1}$

For example $n = 3$, $X = 10$ gives $Area = 2500$

9.5 6.873

9.6 $10\ln(x) + \sin(x) + k$

9.7 Thickness $= a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4$ with $a_0 = -0.0778$, $a_1 = 0.0\,142$, $a_2 = -2.17 \times 10^{-5}$, $a_3 = 1.30 \times 10^{-8}$, $a_4 = -2.86 \times 10^{-12}$
Integration gives $Area = a_0X + (a_1X^2/2) + (a_2X^3/3) + (a_3X^4/4) + (a_4X^5/5) =$ 4180 m^2

9.8 (i) See Fig. b15; (ii) $Q.\Delta z$; (iii) $\sum_{i=1}^{n} Q_i\Delta z = \sum_{i=1}^{n} \frac{y_i}{20}\Delta z$; (iv) $\frac{1}{20}\int_0^{30} y\,dz$; (v) 22.5 kW km^{-2}; (vi) 4444 km^2

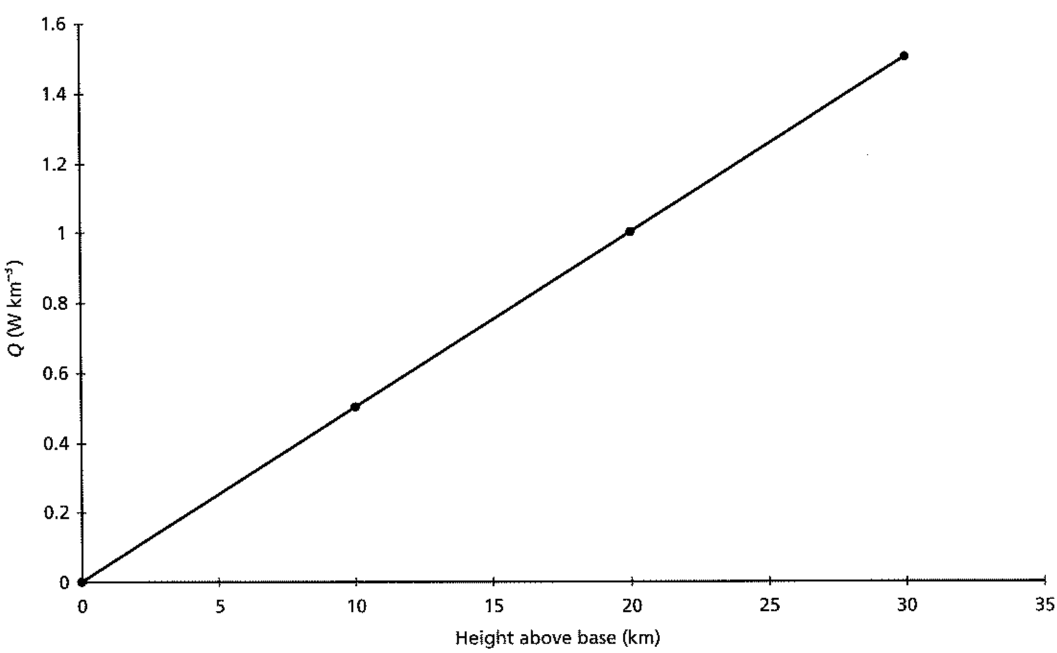

Fig. b15

9.9 (i) $\pi r^2 \Delta z$; (ii) $\sum_{i=1}^{n} \pi r_i^2$; (iii) $\int_{-r_p}^{r_p} \pi r^2 dr = \pi \int_{-r_p}^{r_p} r_e^2[1 - (z^2/r_p^2)]dz$

$= \pi r_e^2\left[2r_p - \frac{2}{3}r_p\right] = \frac{4}{3}\pi r_e^2 r_p$; (iv) 1.083×10^{12} km^3; (v) 1.081×10^{12} km^3

9.10 (i) $t_i \Delta x$; (ii) $\sum_{i=1}^{N} t_i \Delta x$; (iii) Xt_0; (iv) 500 years