Journal of Geoscience Education, v. 48, n. 4, p. 522-532, September 2000 (edits, June 2005)

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Cramer's Rule and the Three-Point Problem

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Introduction

The three-point problem is one of the classic laboratory problems of the undergraduate geology curriculum. Given the elevation of three points on a geologic surface such as a formation contact, what is the attitude (strike and dip) of that surface? A similar question arises in hydrogeology. What is the gradient of the potentiometric surface given its elevation in three wells?

Aside from the fact that the three-point problem arises in real-world applications, geology instructors like the problem because it drives home the meaning of strike and dip.

The three-point problem is also a gateway to some useful mathematics. In this essay, I will discuss two solutions of the three-point problem using Cramer's Rule, an important technique for solving a small number of simultaneous equations. Cramer's Rule is one of the important methodologies of school algebra, but geology students generally do not see an application of simultaneous equations until advanced courses in geological data analysis or geophysics, and, in those courses, instructors want to approach higher-dimension problems using matrix algebra.

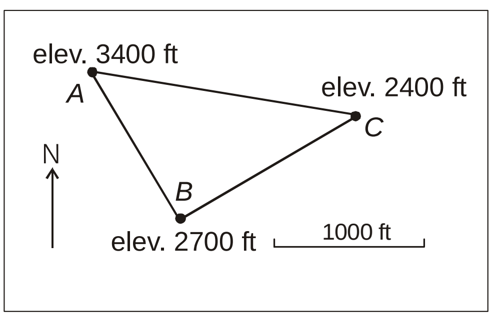

Figure 1. Map showing the location and elevation of three points. Modified slightly from Davis and Reynolds, 1996, Fig. G.7.

The Problem

Figure 1 shows a common presentation of the three-point problem. This example is very similar to the three-point problem discussed in a standard textbook in structural geology. The surface of interest is an unconformity. The elevation of the unconformity is known at three locations (*A*, *B*, and *C*). The horizontal scale is provided. We want to find the strike and dip of the unconformity.

Graphical Solution

The standard approach is graphical. The elevation at *B* is between the elevations at *A* and *C*, so a contour passing through *B* (i.e., the 2700-ft contour) must cross the line segment *AC*. By the definition of strike, the direction of this contour is the strike of the unconformity surface. Thus the first step is to draw this contour (Fig. 2).

We can locate the contour by dividing line segment *AC* into proportional parts according to the elevation differentials. Specifically, the unconformity surface drops 1000 ft between *A* and *C*; 700 ft between *A* and *B′*, and 300 ft between *B′* and *C*. Therefore, *B′* must be 70% of the distance from *A* to *C*. This locates *B′*. So, we draw the line segment *BB′* (Figure 2) and measure the azimuth of the strike with a protractor.

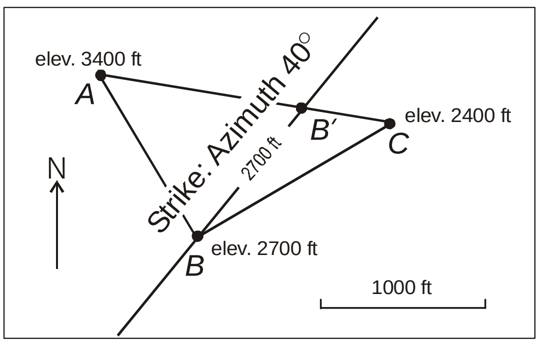

Figure 2. Map showing the location of the line of strike from data in Figure 1.

The second step is to find the dip. To use a fully graphical way of doing this (Davis and Reynolds, 1996), draw a cross-section perpendicular to *BB′*, the line of strike (Figure 3). Then, using a vertical scale equal to the horizontal scale, lay out elevations on the cross section; project the locations of *A*, *B*, and *C* onto the cross-section at the appropriate elevations; and connect the dots. The resulting line segment shows the unconformity in cross-section. Because the cross-section is perpendicular to strike, the included angle is the true dip. So, we measure the angle with a protractor.

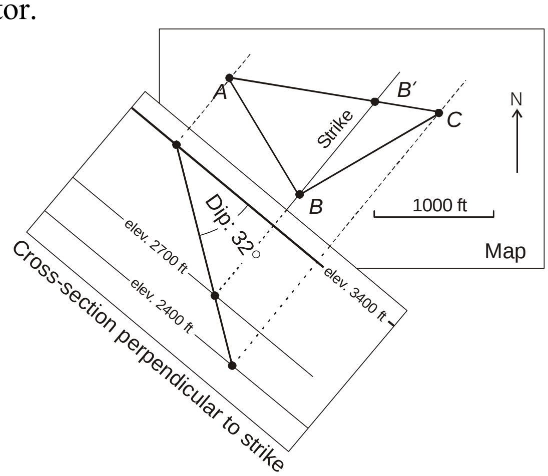

Figure 3. Map and cross-section showing true dip from data in Figure 1 (after Davis and Reynolds, 1996, Fig. G.7)

A slight modification. Sometimes it is easier to find drafting triangles and a ruler than it is to find a protractor. Not to worry, we can easily find the strike and dip by measuring distances and drawing parallels and perpendiculars with the triangles (Figure 4). First draw line segments *AA′* and *CC′* perpendicular to the line of strike. Second, draw right triangle *BDC′* by drawing a vertical line (parallel to the north arrow) through *B* and a horizontal line segment (perpendicular to the north arrow) through *C′*. Then measure the distances *BD*, *DC′*, *AA′*, and *CC′* and find a calculator. The azimuth of strike (θ_{strike}) is

$$\theta_{strike} = \arctan\left(\frac{DC'}{BD}\right). \tag{1}$$

The angle of dip (θ_{dip}) is

$$\theta_{dip} = \arctan\left(\frac{h_A - h_{A'}}{AA'}\right) = \arctan\left(\frac{h_{C'} - h_C}{CC'}\right), \tag{2}$$

where h_A, $h_{A'}$, h_C, and $h_{C'}$ are the elevation at the locations identified by the subscripts.

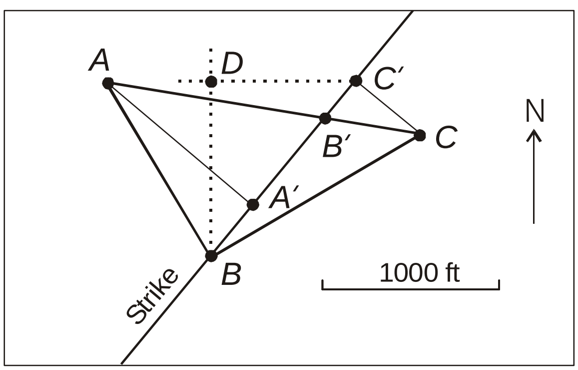

Figure 4. Map showing location of auxiliary lines to calculate strike and dip from arctangents.

Limitations. Three-point problems make good graphical exercises. But graphical solutions take great care. If construction lines are off by only a slight angle, the error in the final answer can be substantial. Moreover, graphical solutions take time. How would you like to have to find the answer to 50 different three-point problems before leaving your desk?

Computational Solutions

There are a number of ways of calculating the strike and dip from three-point data without measuring anything. These algorithms are based on analytical expressions and can be easily programmed. Such programs allow you to solve 50 or more three-point problems in the time that it takes you to enter the data.

Computational solutions generally start with a different presentation of the problem. Figure 5 shows the same problem as Figure 1. The data are in Cartesian form; i.e., the *x*- *y*- and *z*-coordinates of the three points are given.