

KEY

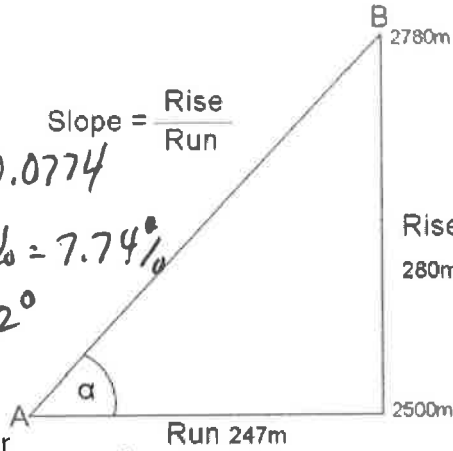
ES302 Quantitative Methods Exercise

Slope Calculation from Contour Lines in a Topographic Map

Slope is the measure of steepness or the degree of inclination of a feature relative to the horizontal plane. Gradient, grade, incline and pitch are used interchangeably with slope. Slope is typically expressed as a percentage, an angle, or a ratio. The average slope of a terrain feature can conveniently be calculated from contour lines on a topographic map. The slope angle expressed in degrees is found by taking the arctangent of the ratio between rise and run.

Example Problem

GRAD CD
 $TAN \theta = \frac{180 \text{ ft}}{2326 \text{ ft}} = 0.0774$
 Slope = $\frac{\text{Rise}}{\text{Run}}$
 GRAD % = $0.0774 \times 100\% = 7.74\%$
 $TAN^{-1}(0.0774) = 4.42^\circ$



$\overline{AB} = 1.02 \text{ in} (31,364) \left(\frac{10 \text{ ft}}{12 \text{ in}} \right) = 2666 \text{ ft}$

$\overline{CD} = (0.89 \text{ in}) (31,364) \left(\frac{10 \text{ ft}}{12 \text{ in}} \right) = 2326 \text{ ft}$

GRAD AB $TAN \theta = \frac{20 \text{ ft}}{2666 \text{ ft}} = 0.008$
 GRAD % = $0.008 \times 100\% = 0.8\%$
 $TAN^{-1}(0.008) = 0.46^\circ$

Gradient (decimal) = Rise / Run

Here for every 1 unit (e.g. meter, foot, etc.) of horizontal travel, there is 1.1336 units of altitude gain. Alternatively for every 0.882 unit horizontal travel, there is one unit of vertical gain. Therefore as a ratio, the gradient would be expressed as (1 in 0.882).

Gradient (percentage) = $1.1336 \times 100 = 113.4\%$

Slope angle is the angle α in the diagram. By definition of tangent in trigonometry: $\tan \alpha = \text{Rise} / \text{Run}$

Therefore having the values for rise and run, value of α in degrees is determined by taking the arctangent (tan-1) of the ratio: $\alpha = \arctan(280/247) = 48.6^\circ$

The distance of travel (distance along the slope or hypotenuse of triangle) is obtained from the Pythagorean theorem equation: $(\text{hypotenuse distance})^2 = (247)^2 + (280)^2$
 Distance along the slope is equal to the square root of the sum shown above = 373m.

Exercise

Examine the attached map sample that is clipped from the Monmouth, Oregon 7.5-minute USGS Quadrangle map. Calculate the following quantities; show all of your math work and unit algebra.

- Fractional Scale 1: 31,364
- Contour Interval (ft) 10 ft = CI
- Distance Pt. A to B (ft) 2666 ft
- Distance Pt. C to D (ft) 2326 ft
- Relief between Pt. A and B 20 feet
- Relief between Pt. C and D 180 feet = 180 ft (10 ft / ft)
- Gradient between Pt. A and B 0.46 degrees
- Gradient between Pt. A and B 0.8 percent
- Gradient between Pt. C and D 4.42 degrees
- Gradient between Pt. C and D 7.74 percent

SCALE: USE SIMILAR RATIO TO CALCULATE

$1.79 \text{ in} = 4600 \text{ ft} \left(\frac{12 \text{ in}}{\text{ft}} \right)$

$\frac{1.79 \text{ in}}{1.79 \text{ in}} = \frac{55,200 \text{ in}}{1.79 \text{ in}}$

1 : 31,364

