UNITS: SI units (Systéme Internationale d'Unités)

Base SI units

length	meter	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

examples of derived units

velocity	m/s
acceleration	m/s^2
area	m^2
density	kg/m^3

Dimensional Analysis $\cdots$ Do the units work out ?

In math class we see quadratic equations of the form

$$y = ax^2 + bx + c$$

all the time and never question its validity. In physics, every quantity has units associated with it (the exception being the ratio of two expressions having the same units). Dimensional analysis allows us to check if the mathematical expression we are using in dimensionally correct. In a short while you'll see the following equation:

$$x(t) = \frac{1}{2}at^2 + v_0 t + x_0$$

where $x(t)$ and x_0 are measured in meters (m), t in seconds (s), the initial velocity v_0 in meters/sec (m/s), and acceleration a in m/s^2. By substituting in the appropriate units for each quantity we observe that

$$m = (\frac{m}{s^2}) * (s)^2 + (\frac{m}{s}) * s + m = m + m + m,$$

verifying that the expression is indeed dimensionally correct.

Had we mistakenly put the squared on the second occurrence of time (t) in the equation rather than the first we'd have

$$x(t) = \frac{1}{2}at + v_0 t^2 + x_0,$$

which would lead to the following dimensionalities:

$$m = (\frac{m}{s^2}) * (s) + (\frac{m}{s}) * (s)^2 + m = \frac{m}{s} + m * s + m.$$

Note that the addition operation is now ill-defined; you can only add quantities of the same type. Thus, checking equations using dimensional analysis helps to catch algebraic mistakes you might make during mathematical manipulations.

Another equation we'll derive when we begin studying projectile motion is known as the range equation:

$$R = \frac{v_0^2 \sin(2\theta_0)}{g},$$

where R is the horizontal distance traveled (the Range), v_0 is the initial velocity, θ_0 is the angle it is launched at relative to the horizontal, and g is the acceleration due to gravity. Inserting the units demonstrates that

$$m = \frac{(\frac{m}{s})^2 * 1}{(\frac{m}{s^2})} = m.$$

Note that in our units check I've inserted "1" for the units of the sine function, trigonometric functions are pure numbers, they have no units associated with them.

Powers of ten

Usually we'll use the power of ten that is most convenient for the problem at hand. For a problem involving the thickness of a cell wall, length measurements will often be in terms of nanometers (1 nm $= 10^{-9}$ m), while for a car traveling down the highway a more appropriate unit of measurement might be the kilometer (1 km $= 10^{3}$ m). Some commonly used unit prefixes and powers of ten:

pico	p	10^{-12}	tera	T	10^{12}
nano	n	10^{-9}	giga	G	10^{9}
micro	μ	10^{-6}	mega	M	10^{6}
milli	m	10^{-3}	kilo	k	10^{3}
centi	c	10^{-2}			

Unit Conversions $\cdots$ always multiply by one!

Basic idea: since 1 minute = 60 seconds, divide each side by 60 seconds to get:

$$\frac{1 \text{ minute}}{60 \text{ seconds}} = \frac{60 \text{ seconds}}{60 \text{ seconds}} = 1$$

We could just have easily divided by 1 minute to arrive at:

$$\frac{60 \text{ seconds}}{1 \text{ minute}} = \frac{1 \text{ minute}}{1 \text{ minute}} = 1.$$

Since multiplying by 1 leaves the value of a quantity unchanged, we can accomplish our conversions from one system of units to another by multiplying by the "appropriate form" of 1.

Example: lets convert 55 miles/hour to meters/second.

$$\frac{55\ \text{mi}}{1\ \text{hr}}=\left(\frac{55\ \cancel{\text{mi}}}{1\ \cancel{\text{hr}}}\right)\left(\frac{1\ \cancel{\text{hr}}}{60\ \cancel{\text{min}}}\right)\left(\frac{1\ \cancel{\text{min}}}{60\ \text{sec}}\right)\left(\frac{5280\ \cancel{\text{ft}}}{1\ \cancel{\text{mi}}}\right)\left(\frac{12\ \cancel{\text{in}}}{1\ \cancel{\text{ft}}}\right)\left(\frac{2.54\ \cancel{\text{cm}}}{1\ \cancel{\text{in}}}\right)\left(\frac{1\ \text{m}}{100\ \cancel{\text{cm}}}\right)$$

$$\frac{55\ \text{mi}}{1\ \text{hr}} = \left(\frac{55 * 5280 * 12 * 2.54}{60 * 60 * 100}\right)\left(\frac{\text{m}}{\text{s}}\right)$$

$$\text{Thus } \frac{55\ \text{mi}}{1\ \text{hr}} = 24.6\frac{\text{m}}{\text{s}}.$$

Another example: convert 2.7 grams/cm^3 to kg/m^3.

$$\frac{2.7\ \text{gr}}{1\ \text{cm}^3} = \left(\frac{2.7\ \cancel{\text{gr}}}{1\ \cancel{\text{cm}^3}}\right)\left(\frac{1\ \text{kg}}{10^3\ \cancel{\text{gr}}}\right)\left(\frac{10^2\ \cancel{\text{cm}}}{1\ \text{m}}\right)^3$$

$$\text{Thus } \frac{2.7\ \text{gr}}{1\ \text{cm}^3} = \left(\frac{2.7 * 10^6}{10^3}\right)\left(\frac{\text{kg}}{\text{m}^3}\right) = 2.7 * 10^3\left(\frac{\text{kg}}{\text{m}^3}\right)$$