Answer Key

Problem Set 2

Reservoir Analysis

Assume $g = 9.81\ m/sec^2$

1. *A fluid in an aquifer is 4.0 m above a reference datum, the fluid pressure is 2,400 N/m^2 and the flow velocity is 1.0×10^{-5} m/sec. If the fluid density is $1.01 \times 10^3\ kg/m^3$,*

   A. *What is the total energy per unit mass?*

   B. *What is the total energy per unit weight?*

Solution:

Using Equation 5-6

A. $E_{tm} = v^2/2 + gz + P/\rho$

$$\frac{(1.0 \times 10^{-5})^2}{2} \frac{m^2}{s^2} + 9.81 \frac{m}{s^2} \cdot 4.0\ m + \frac{2400 \frac{N}{m^2}}{1.01 \times 10^3\ kg/m^3}$$

$E_{tm} = 42\ m^2/s^2$

B.

$$E_{tw} = \frac{E_{tm}}{g} = \frac{41.62 \frac{m^2}{s^2}}{9.81 \frac{m}{s^2}} = \underline{4.2\ m}$$

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2. *A fluid in an aquifer is 31.5 m above a reference datum, the fluid pressure is 3,750 N/m^2, and the flow velocity is $1.35 \times 10^{-4}\ m/sec^2$. If the fluid density is $0.999 \times 10^3\ kg/m^3$,*

   A. *What is the total energy per unit mass?*

   B. *What is the total energy per unit weight?*

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Solution:

Using Equation 5-6:

A. $$E_{tm} = v^2/2 + gz + P/\rho$$

$$= \frac{(1.35 \times 10^{-4})^2}{2} \frac{m^2}{s^2} + 9.81 \frac{m}{s^2} \cdot 31.5\,m + \frac{3750 \frac{N}{m^2}}{0.999 \times 10^3 \frac{kg}{m^3}}$$

$$\underline{E_{tm} = 313\ m^2/s^2}$$

B.

$$E_{tw} = \frac{E_{tm}}{g} = \frac{312.77 \frac{m^2}{s^2}}{9.81 \frac{m}{s^2}} = \underline{31.9\ m}$$

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3. *A piezometer is screened 274.3 m above mean sea level. The point water pressure head in the piezometer is 23.4 m and the water in the aquifer is fresh at a temperature of 20⁰ C.*

   A. *What is the total head in the aquifer at the point where the piezometer is screened?*

   B. *What is the fluid pressure in the aquifer at the point where the piezometer is screened?*

<u>Solution</u>:

From Equation 5-11 and Figure 5.2:

A. $h = z + h_\rho = 273.4\ m + 23.4\ m$

$\underline{h = 296.8\ m}$

B. From Equation 5-10:

$P = \rho g h_\rho$

ρ_W at $20^0C = 0.998203\ g/cm^3$

$0.998203\ g/cm^3 \times 1\ kg/1000\ g \times 10^6\ cm^3/m^3 = 998.20\ kg/m^3$

Problem 5-3 (cont.)

$$P = 998.2\ kg/m^3 \cdot 9.81\ m/s^2 \cdot 23.4\ m$$

$$P = 2.29 \times 10^5\ kg/m \cdot s^2$$

$$= 2.29 \times 10^5\ N/m^2$$

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4. *A piezometer point is 23 m above mean sea level. The fluid pressure in the aquifer at that point is $6.45 \times 10^6\ N/m^2$. The aquifer has fresh water at a temperature of 13^oC.*

   A. *What is the point water pressure head?*

   B. *What is the total head?*

Solution:

From Equation 5-10:

A. $P = \rho g h_p$, or $h_p = P/\rho g$

ρ_w at $13^oC = 999.38\ kg/m^3$

$$h_p = \frac{6.45 \times 10^6\ \frac{N}{m^2}}{999.38\ \frac{kg}{m^3} \cdot 9.81\ \frac{m}{s^2}}$$

$$h_p = 658\ m$$

B. From Equation 5-16:

$$h = z + h_p$$

$$= 23\ m + 657.9\ m$$

$$h = 681\ m$$

5. *A piezometer in a saline water aquifer has a point water pressure head of 15.23 m. If the water has a density of 1029 kg/m^3 and is at a field temperature of 21^{o}C, what is the equivalent fresh water pressure head?*

Solution:

From Equation 5-10:

$P = \rho g h_\rho$

$P = 1029\ kg/m^3 \cdot 9.81\ m/s^2 \cdot 15.23\ m$

$\underline{P = 1.54 \times 10^5\ N/m^2}$

Now assume fresh water:

$P = \rho g h_\rho$ and $h_\rho = P/\rho g$

ρ_{fresh} at 21^{o}C = 997.99 kg/m^3

$$h_\rho = \frac{1.54 \times 10^5\ \frac{N}{m^2}}{997.99\ \frac{kg}{m^3} \cdot 9.81\ \frac{m}{s^2}}$$

$\underline{h_\rho = 15.7\ m}$

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6. *The fluid pressure in the screen of a piezometer in a saline aquifer is 4.532 x 10^5 N/m^2. The fluid density is 1073 kg/m^3 and the temperature is 12^{o}C. The elevation of the piezometer screen is 1048.54 m a.s.l.*

   A. *Compute the point water pressure head.*

   B. *Compute the fresh water pressure head.*

   C. *Find the total fresh water head.*

Solution:

$P = \rho g h_\rho$ and $h_\rho = P/\rho g$

Problem 5-6 (cont.)

A.

$$h_{\rho_{saline}} = \frac{4.532 \times 10^5 \frac{N}{m^2}}{1073 \frac{kg}{m^3} \cdot 9.81 \frac{m}{s^2}}$$

$$\underline{h_{\rho_{saline}} = 43.1 \text{ m}}$$

B.

$$h_{\rho_{fresh}} = P/\rho_{fresh}g$$

$$\rho_{fresh} = 999.498 \text{ kg/m}^3$$

$$h_{\rho_{fresh}} = \frac{4.532 \times 10^5 \frac{N}{m^2}}{999.498 \frac{kg}{m^3} \cdot 9.81 \frac{m}{s^2}}$$

$$\underline{h_{\rho_{fresh}} = 46.2 \text{ m}}$$

C. $h = z + h_{\rho_{fresh}}$

$h = 1048.54 \text{ m} + 46.2 \text{ m} = \underline{1094.7 \text{ m}}$

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7. *A sand aquifer has a median pore diameter of 0.2 mm. The fluid density is 1.003×10^3 kg/m^3 and the fluid viscosity is 1.15×10^{-3} N-sec/m^2. If the flow rate is 0.0016 m/sec. is Darcy's law valid? What is the reason for your answer?*

Solution:

From Equation 5-20:

$R = \rho v d/\mu$

$$= \frac{1.003 \times 10^3 \frac{kg}{m^3} \cdot 0.0016 \frac{m}{s} \cdot 0.00002 \text{ m}}{1.15 \times 10^{-3} \frac{N-s}{m^2}}$$

R = 0.3

Reynolds number is less than 10; Darcy's law is valid.

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8. *An aquifer has a hydraulic conductivity of 123 ft/day, an effective porosity of 27% and is under a hydraulic gradient of 0.0003.*

A. Compute the Darcy flux.

B. Compute the average linear velocity.

C. The water temperature was 12°C and the mean pore diameter was 0.33 mm. Was it permissible to use Darcy's law under these circumstances? What is the reason for your answer?

<u>Solution</u>:

From Equation 5-18:

$$Q = -KA\ (dh/dl) \quad \text{and} \quad Q/A = v = -K\ (dh/dl)$$

A. $v = 123 \text{ ft/d} \cdot 0.0003$

<u>$v = 0.04$ ft/d</u>

From Equation 5-24:

B. $V_x = (K/n_e)\ (dh/dl)$

<u>$V_x = 0.1$ ft/d</u>

convert: $0.1 \frac{\text{ft}}{\text{d}} \times 12 \frac{\text{in}}{\text{ft}} \times 2.54 \frac{\text{cm}}{\text{in}} \times \frac{1 \text{ d}}{24 \text{ h}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ sec}}$

$V_x = 4.82 \times 10^{-5}$ cm/s

C. calculate Reynolds number:

$R = \rho vd/\mu$ (Equation 5-20)

$d = 0.33$ mm $= 0.033$ cm

$\rho_{12^oC} = .99498\ g/cm^3$

$\mu_{12^oC} = 0.012363$ poise = g/sec-cm

$$R = \frac{0.999498\ \frac{g}{cm^3} \cdot 4.82 \times 10^{-5}\ \frac{cm}{s} \cdot 0.033cm}{0.012363\ \frac{g}{sec\text{-}cm}}$$

R = 0.0001, which is less than 10; Darcy's law is valid.

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9. *A confined aquifer is 10 feet thick. The potentiometric surface drops 0.54 foot between two wells that are 792 feet apart. The hydraulic conductivity is 21 feet per day and the effective porosity is 0.17.*

A. *How many cubic feet per day are moving through a strip of the aquifer that is 10 feet wide?*

B. *What is the average linear velocity?*

<u>Solution</u>:

From Equation 5-56:

A. $q' = Kb\ (dh/dl)$ where $K = 21$ ft/d
$b = 10$ ft
$dh/dl = 0.54$ ft/792 ft

$q' = 21\ ft/d \cdot 10\ ft \cdot 0.54\ ft/792\ ft$

$Q = 0.14\ ft^2/d \times 10\ ft$

<u>$Q = 1.4\ ft^3/d$</u> moving through a 10 foot-wide strip

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B. From Equation 5-24:

$V_x = Q/An_e$

where $A = 10 \text{ ft} \times 10 \text{ ft} \times 100 \text{ ft}^2$

$q = 1.4 \text{ ft}^3/\text{d}$

$$V_x = \frac{1.4 \text{ ft}^3/\text{d}}{100 \text{ ft}^2 \cdot 0.17}$$

$V_x = 0.082 \text{ ft/d}$

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10. *A confined aquifer is 24.5 m thick. The potentiometric surface drops 1.23 m between two wells that are 1023 m apart. If the hydraulic conductivity of the aquifer is 44 m/day, how many cubic meters of flow are moving through the aquifer per unit width?*

Solution:

From Equation 5-56:

$q' = Kb\ (dh/dl)$

$q' = 44 \text{ m/d} \cdot 24.5 \text{ m} \cdot 1.23 \text{ m}/1023 \text{ m}$

$q' = 1.3 \text{ m}^2/\text{d}$ per unit width

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11. *An unconfined aquifer has a hydraulic conductivity of 1.7×10^{-3} cm/sec. There are two observation wells 328 feet apart. Both penetrate the aquifer to the bottom. In one observation well the water stands 24.6 feet above the bottom and in the other it is 20.0 feet above the bottom.*

A. *What is the discharge per 100-foot-wide strip of the aquifer in ft^3/day?*

B. *What is the water-table elevation at a point midway between the two observation wells?*

Solution:

From Equation 5-60:

$$q' = K\left(\frac{h_1^2 - h_2^2}{2L}\right)$$

where $h_1 = 24.6$ ft

$h_2 = 20.0$ ft

$L = 328$ ft

$K = 1.7 \times 10^{-3}$ cm/s

$= 4.8$ ft/d

A.

$$q' = 4.8\text{ft/d}\left(\frac{24.6^2\,\text{ft}^2 - 20.0^2\,\text{ft}^2}{2\,(328)\,\text{ft}}\right)$$

$q' = 1.5$ ft^2/day per unit width

1.5 ft^2/d $\cdot$ 100 ft = 150 ft^3/d

B. From Equation 5-72:

$$h = \sqrt{h_1^2 - \frac{(h_1^2 - h_2^2)\,x}{L}}$$

where $h_1 = 24.6$ ft
$h_2 = 20.0$ ft
$L = 328$ ft
$x = 164$ ft

$$h = \left[24.6^2\,\text{ft}^2 - \frac{(24.6^2\,\text{ft}^2 - 20^2\,\text{ft}^2)\cdot 164\ \text{ft}}{328\ \text{ft}}\right]^{1/2}$$

$h = 22.4$ ft