ANSWER KEY — PROBLEM SET 1

RESERVOIR ANALYSIS

1. *What is the weight in newtons of an object with a mass of 32.1 kg?*

Solution:

From Equation 4-3:

Mass = 32.1 kg

$w = mg$ where $g = 9.80\ m/sec^2$

$w = 32.1\ kg \cdot 9.80\ m/sec^2$

$w = 314.6\ kg\text{-}m/sec^2 = \underline{315\ N}$

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2. *What is the weight in pounds of an object with a mass of 12.4 slugs?*

Solution:

$m = 12.4$ slugs

$w = m \cdot g$ where $g = 32.17\ ft/g^2$

$= 12.4\ slug \cdot 32.17\ ft/sec^2$

$w = 398.9\ slug \cdot ft/sec^2 = 399\ lb$

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3. *If an object has a mass of 55.3 kg and a volume of 0.33 m^3,*

   A. *What is its density?*

   B. *What is its specific weight?*

   C. *Is the object more or less dense than water?*

Solution:

From Equation 4-4:

$\rho = m/V$ $m = 55.3\ kg$

$V = 0.33\ m^3$

Problem 4-3 (cont.)

A. $\rho = 55.3/0.33$

$= 168\ kg/m^3 = \underline{170\ kg/m^3}$

B. From Equation 4-5:

$\gamma = w/V$ and $w = mg$, then $\gamma = \rho g$

$\rho = 168\ kg/m^3$, $g = 9.80\ m/sec^2$

$\gamma = 1646\ kg\text{-}m/sec^2\text{-}m^3 = \underline{1600\ N/m^3}$

C. Assume water has a density of $1\ g/cm^3 = 1000\ kg/m^3$; the object is less dense than water.

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4. *If an object has a mass of 723 kg and a volume of .56 m^3,*

A. *What is its density?*

B. *What is its specific weight?*

C. *Is it more or less dense than water?*

<u>Solution</u>:

A. $\rho = m/V$

$\rho = 723\ kg/0.56\ m^3$

$\rho = 1291\ kg/m^3 = \underline{1300\ kg/m^3}$

B. $\gamma = \rho g$

$= 723\ kg/0.56\ m^3 \cdot 9.80\ m/sec^2$

$\gamma = 12653\ N/m^3 = \underline{13{,}000\ N/m^3}$

C. Assume $\rho_{water} = 1000\ kg/m^3$; object is more dense than water.

5. *The hydraulic conductivity of a silty sand was measured in a laboratory permeameter and found to be 1.36 x 10^{-5} cm/sec at 25^{o} C. What is the intrinsic permeability in cm^2? Refer to appendix 14 to get values for density and viscosity.*

<u>Solution</u>:

$K = 1.36 \times 10^{-5}$ cm/s @ $25^{o}C$

From Equation 4-18:

$K = K_i \rho g/\mu$

Rearranging

$K_i = K\mu/\rho g$

at $25^{o}C$: $\rho = 0.997044$ g/cm^3

$\mu = 0.008937$ poise (g/cm-sec)

$g = 980$ cm/sec^2

$$K_i = \frac{(1.36 \times 10^{-5}\ cm/s)\ (0.008937\ g/cm\text{-}s)}{(0.997044\ g/cm^3)\ (980\ cm/sec^2)}$$

$$K_i = 1.24 \times 10^{-10}\ \frac{cm\text{-}g/cm\text{-}s^2}{g\text{-}cm/cm^3\ s^2}$$

$$\underline{K_i = 1.24 \times 10^{-10}\ cm^2}$$

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6. *The hydraulic conductivity of a coarse sand was measured in a laboratory permeameter and found to be 7.92 x 10^{-3} cm/sec at $25^{o}C$. What is the intrinsic permeability. Refer to appendix 14 to obtain values for density and viscosity.*

<u>Solution</u>:

$K = 7.92 \times 10^{-3}$ cm/s @ $25^{o}C$

$K = K_i \rho g/\mu$ or $K_i = K\mu/\rho g$

at $25^{o}C$: $\rho = 0.997044$ g/cm^3

$\mu = 0.008937$ g/cm-s

$g = 980$ cm/s^2

$$K_i = \frac{7.92 \times 10^{-3} \cdot 0.008937}{0.997044 \cdot 980} \left[\frac{\frac{cm}{s} \cdot \frac{g}{cm\text{-}s}}{\frac{g}{cm^3} \cdot \frac{cm}{s^2}} \right]$$

$K = 7.24 \times 10^{-8}\ cm^2$

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7. *A constant head permeameter has a cross-sectional area of 156 cm^2. The sample is 18 cm long. At a head of 5 cm, the permeameter discharges 50 cm^3 in 193 seconds.*

   A. *What is the hydraulic conductivity in cm/sec and ft/day?*

   B. *What is the intrinsic permeability if the hydraulic conductivity was measured at 15°C?*

Solution:

from Equation 4-22: $K = VL/Ath$ where
$V = 50\ cm^3$
$t = 193\ s$
$L = 18\ cm$
$A = 156\ cm^2$
$h = 5\ cm$

A.

$$K = \frac{50 \cdot 18}{156 \cdot 193 \cdot 5} \left[\frac{cm^3 \cdot cm}{cm^2 \cdot s \cdot cm} \right]$$

$K = 6 \times 10^{-3}\ cm/s$ or 17 ft/d

B. at 15°C, $\rho = 0.999099\ g/cm^3$

$\mu = 0.011404\ g/cm\text{-}s$

$g = 980\ cm/s^2$

$$K_i = \frac{K\mu}{\rho g} = \frac{6.0 \times 10^{-3} \cdot 0.011404}{0.999099 \cdot 980} \left[\frac{\frac{cm}{s} \cdot \frac{g}{cm\text{-}s}}{\frac{g}{cm^3} \cdot \frac{cm}{s^2}}\right]$$

$\underline{K_i = 7 \times 10^{-8}\ cm^2}$

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8. *A constant head permeameter has a cross-sectional area of 225 cm^2. The sample is 25 cm long. At a head of 15 cm, the permeameter discharges 50 cm^3 in 456 seconds.*

   A. *What is the hydraulic conductivity in cm/sec and ft/day?*

   B. *What is the intrinsic permeability if the hydraulic conductivity was measured at 20°C?*

Solution:

from Equation 4-22: $K = VL/Ath$ where
$V = 50\ cm^3$
$t = 456\ s$
$L = 25\ cm$
$A = 225\ cm^2$
$h = 15\ cm$

A.

$$K = \frac{50 \cdot 25}{225 \cdot 456 \cdot 15} \left[\frac{cm^3 \cdot cm}{cm^2 \cdot s \cdot cm}\right]$$

$\underline{K = 8.1 \times 10^{-4}\ cm/s}$ or $\underline{2.3\ ft/d}$

B. at 20°C, $\rho = 0.998203\ g/cm^3$

$\mu = 0.010050\ g/cm\text{-}s$

$g = 980\ cm/s^2$

$$K_i = \frac{K\mu}{\rho g} = \frac{8.1 \times 10^{-4} \cdot 0.010050}{0.998203 \cdot 980} \left[\frac{\frac{cm}{s} \cdot \frac{g}{cm\text{-}s}}{\frac{g}{cm^3} \cdot \frac{cm}{s^2}} \right]$$

$$\underline{K_i = 8.3 \times 10^{-9}\ cm^2}$$

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9. *An aquifer has a specific yield of 0.19. During a drought period the following declines in the water table were noted:*

Area	Size	Decline
A	14 mi^2	2.75 feet
B	7 mi^2	3.56 feet
C	28 mi^2	5.42 feet
D	33 mi^2	7.78 feet

What was the total volume of water represented by the decline in the water table?

Solution:

From Equation 4-34, $V_w = SA\ \Delta h$, where

$S = 0.19$
$A_A = 14\ mi^2 = 3.9 \times 10^8\ ft^2$
$A_B = 7\ mi^2 = 2.0 \times 10^8\ ft^2$
$A_C = 28\ mi^2 = 7.8 \times 10^8\ ft^2$
$A_D = 33\ mi^2 = 9.2 \times 10^8\ ft^2$

$\Delta h_A = 2.75\ ft$
$\Delta h_B = 3.56\ ft$
$\Delta h_C = 5.42\ ft$
$\Delta h_D = 7.78\ ft$

Area A: $V_w = 0.19 \cdot 3.9 \times 10^8\ ft^3 \cdot 2.75\ ft$

$V_{w_A} = 2.0 \times 10^8\ ft^3$

Area B: $V_{w_B} = 1.3 \times 10^8\ ft^3$

Problem 4-9 (cont.)

Area C: $V_{w_C} = 8.0 \times 10^8 \text{ ft}^3$

Area D: $V_{w_D} = 1.4 \times 10^9 \text{ ft}^3$

Total = $\Sigma V = 2.0 \times 10^8 + 1.3 \times 10^8 + 8.0 \times 10^8 + 1.4 \times 10^9 \text{ ft}^3$

$\underline{V_w = 2.5 \times 10^9 \text{ ft}^3}$

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10. *A confined aquifer has a specific storage of 3.5×10^{-6} ft^{-1} and a thickness of 200 feet. How much water would it yield if the water declined an average of 2.5 feet over a circular area with a radius of 375 feet?*

<u>Solution</u>:

From Equation 4-34: $V_w = SA\,\Delta h$

$S_s = 3.5 \times 10^{-6} \text{ ft}^{-1}$

$b = 200 \text{ ft}$

$S = S_s b = 7.0 \times 10^{-4}$

$A = \pi r^2 = (3.14)\ (375 \text{ ft})^2$

$A = 4.42 \times 10^5 \text{ ft}^3$

$\Delta h = 2.5 \text{ ft}$

$V_w = 7.0 \times 10^{-4} \cdot 4.42 \times 10^5 \cdot 2.5 \text{ ft}$

$\underline{V_w = 770 \text{ ft}^3}$

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11. *A confined aquifer has a specific storage of 7.5×10^{-6} m^{-1} and a porosity of 0.3. The compressibility of water is 4.6×10^{-10} m^2/N. What is the compressibility of the aquifer skeleton?*

Solution:

From Equation 4-31:

$$S_s = \rho_w g\ (\alpha + n\beta)$$

$$S_s = \rho_w g\alpha + \rho_w g n\beta$$

$$\alpha = \frac{S_s - \rho_w g\, n\, \beta}{\rho_w g}$$

$$\alpha = \frac{7.5 \times 10^{-6} - (1000 \cdot 9.80 \cdot 0.3 \cdot 4.6 \times 10^{-10})}{1000 \cdot 9.80}$$

$$\underline{\alpha = 6 \times 10^{-10}\ m^2/N}$$

$$\text{Units:}\quad \frac{\frac{1}{m} - \left[\frac{kg}{m^3} \cdot \frac{m}{s^2} \cdot \frac{m^2}{\frac{kg - m}{s^2}}\right]}{\frac{kg}{m^3} \cdot \frac{m}{s^2}} = \frac{\frac{1}{m}}{\frac{kg}{m^2 s^2}} = \frac{m s^2}{kg}$$

$$\frac{m - s^2}{kg} = \frac{m^2}{N}$$

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12. *A confined aquifer has a specific storage of 8.8 x 10^{-6} m^{-1} and a porosity of 0.25. The compressibility of water is 4.6 x 10^{-10} m^2/N. What is the compressibility of the aquifer skeleton?*

Solution:

From Equation 4-31

$$S_s = \rho_w\ g\ (\alpha + n\beta)$$

$$S_s = \rho_w g\alpha + \rho_w g n\beta$$

Problem 4-12 (cont.)

$$\alpha = \frac{S_s - \rho_w g n \beta}{\rho_w g}$$

$$\alpha = \frac{8.8 \times 10^{-6} - (1000 \cdot 9.80 \cdot .25 \cdot 4.6 \times 10^{-10})}{1000 \cdot 9.80}$$

$$\alpha = \underline{7.8 \times 10^{-10}\ m^2/N}$$

See problem 4-11 for units.

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13. *An aquifer has three different formations. Formation A has a thickness of 30 feet and a hydraulic conductivity of 7.0 feet per day. Formation B has a thickness of 15 feet and a conductivity of 78 feet per day. Formation C has a thickness of 22 feet and a conductivity of 17 feet per day. Assume that each individual formation is isotropic and homogeneous. Compute both the overall horizontal and vertical conductivity.*

Solution:

Formation	K	b
A	7.0 ft/d	30 ft
B	78 ft/d	15 ft
C	17 ft/d	22 ft

d = total thickness = 67 feet

From Equation 4-40:

$$K_h\ avg = \sum_{m=1}^{n} \frac{K_{hm} b_m}{b}$$

$$K_h\ avg = \frac{7.0 \cdot 30}{67} + \frac{78 \cdot 15}{67} + \frac{17 \cdot 22}{67}$$

$$\underline{K_h\ avg = 26\ ft/d}$$

Problem 4-13 (cont.)

From Equation 4-41:

$$K_v \text{ avg} = \frac{b}{\sum_{m=1}^{n} \frac{b_m}{K_{vm}}}$$

$$= \frac{67}{\frac{30}{7} + \frac{15}{78} + \frac{22}{17}}$$

$$\underline{K_v \text{ avg} = 12 \text{ ft/d}}$$

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14. *Use the Hazen method to estimate the hydraulic conductivity of the sediments graphed on Figures 4.4, 4.5, and 4.30.*

Solution:

From Equation 4-19, $K = C(d_{10})^2$

Figure 4.4 silty fine to coarse sand, $d_{10} = 0.017$ mm, $C \cong 90$

$= 0.0017$ cm

$K = 90\ (.0017)^2$

$K = 2.6 \times 10^{-4}$ cm/s

Figure 4.5 well sorted fine sand, $d_{10} = 0.16$ mm, $C \cong 80$

$= 0.016$ cm

$K = 80\ (0.016)^2$

$K = 2 \times 10^{-2}$ cm/s

Figure 4.30

$d_{10} = 0.16$ mm, $C \cong 100$

$= 0.016$ cm

$K = 100\ (0.016)^2$

$K = 2.6 \times 10^{-2}$ cm/s

15. *Determine the effective grain size and uniformity coefficient of the sediments graphed on Figure 4.30.*

<u>Solution</u>:

Effective grain size = d_{10} = 0.16 mm

Uniformity coefficient = d_{60}/d_{10} = 0.32 mm/0.16 mm = 2.0

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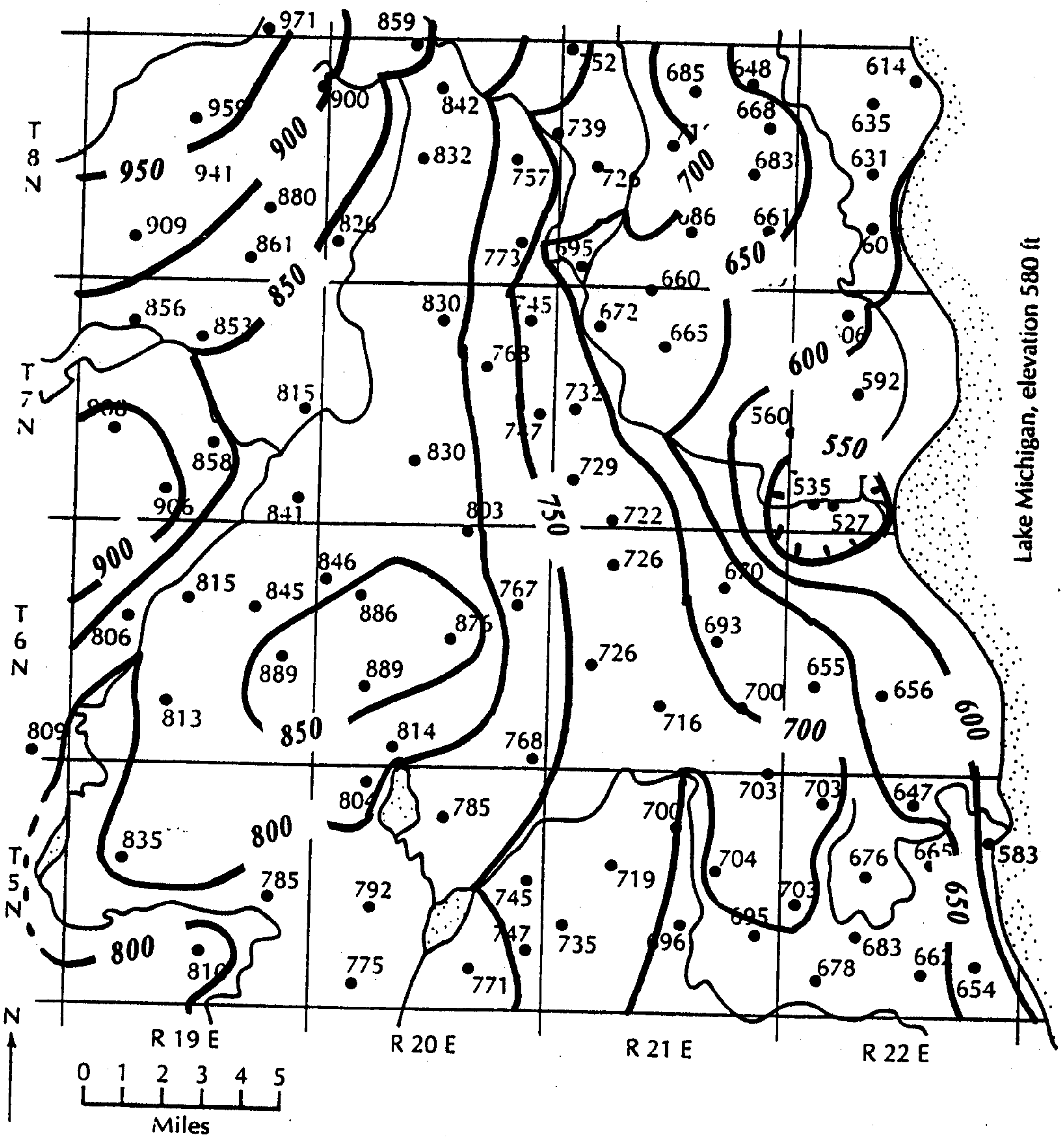
16. *Figure 4-31 is a map showing the ground-water elevation in wells screened in an unconfined aquifer at Milwaukee, Wisconsin. The aquifer is in good hydraulic connection with Lake Michigan, which has a surface elevation of 580 feet above sea level. Lakes and streams are also shown on the map.*

    A. *Make a water table map with a contour interval of 50 feet, starting at 550 feet.*

    B. *Why do you suppose that ground-water levels are below the Lake Michigan surface elevation in part of the area?*

Solution:

Figure 4-31. Base map for question 16.



B. Water levels are below Lake Michigan because of pumping from the aquifer.

Answers to Problem 17

Part A. pH $=3.8$, pH$=6$, pH$=4.5$ pH$=13$, pH$=12.3$

Part B. $[H+]=10^{-3.6}$, $[H+]=-10^{-13.7}$, $[H+]=10^{-9}$, $[H+]=10^{-4.5}$, $[H+]=10^{-6.3}$