Principles of Ground-Water Flow

I. Introduction

A. Energy Distribution in Ground Water
1. Mechanical
2. Thermal
3. Chemical

B. Energy Disequilibrium
1. Spatially variable distribution of energy in system
   a. Ground water flow from high energy to low energy
   b. Dissipation of energy differentials to static equilibrium

C. Flow Mechanisms

1. Chemical: concentration gradients of dissolved solutes in ground water

   a. Diffusion: process by which dissolved ionic and molecular species in water move from areas of higher concentration to areas of lower concentration.

      (1) Advection: process by which moving ground water carries with it dissolved solutes.
         (a) solutes in transport by flowing water

      (2) Dispersion: mechanical and chemical distribution of chemical solute in porous media acting to dilute the solute and lower its concentration

      (3) Retardation: rate of solute transport slower than rate of ground water advection rate
         (a) result of chemical interaction at molecular/atomic scale

2. Thermal Flux: transport of thermal energy along thermal gradients from high energy to low energy.

   a. Simplifying assumption: ground water systems at constant temperature

   b. Area of concern: geothermally active ground water systems and/or radioactive heat sources.

3. Mechanical Transport of Ground Water

   a. External Forces Acting on Ground Water
      (1) Force of Gravity (F = mg)
      (2) External pressure

         (a) Atmospheric Pressure
            i) Water Table Aquifers

(b) Hydrostatic Pressure: pressure exerted by overlying water and overlying rock strata

(3) Molecular Attraction
(a) Dipolar nature of water molecule
(b) Causes water to adhere to solid surfaces
(c) High surface tension.... capillarity

D. Resistance to Ground Water Flow

1. Advective Friction: frictional resistance to flow between moving water molecule and surface of solid particle along pore-throat opening

2. Fluid Viscosity: inherent resistance of fluid to flow
a. internal molecular attraction of fluid itself

II. Mechanical Energy

A. Kinetic Energy: energy of motion

$E_k = (0.5)mv^2$ where E_k = kinetic energy, m = mass, v = velocity

B. Gravitational Potential Energy: energy of position

$E_p = mgz$ where m = mass, g = acceration of gravity,
z = elevation of center of gravity of fluid above reference elevation

C. Energy of Fluid Pressures: pressure force exerted on a fluid mass by surrounding fluid acting upon it.

$P = F/A$ P = pressure, F = force, A = area

D. Total Energy of Fluid Mass System (Sum of Three Components: kinetic, potential, pressure)

assuming density = mass/volume, then density (p) = mass for a unit volume of fluid (i.d. V = 1), substituting p for m in above equations...

$$E_{tv} = E_k + E_p + P = 1/2\ pv^2 + pgz + P$$

where E_{tv} = total energy per unit volume, v = velocity, g = gravity, z = height above reference datum, P = fluid pressure, p = density *by dividing out by p..

**** $E_{tm} = (V^2/2) + gz + (P/p)$ = "Bernoulli Equation"

where E_{tm} = total energy per unit mass (units = L^2/T_2)

**** $E_{tw} = (E_{tm})/g$ = total energy per unit weight (units = L)

E. Ideal Fluid Conditions

1. Steady Flow Conditions: mechanical conditions do not change with time
2. Friction = 0
3. Fluid incompressible, i.e. maintains constant density

F. Realistic Fluid Conditions

1. Unsteady flow conditions
2. Friction = force of resistance
3. Fluids are compressible, hence density changes with conditions

G. Definition of Hydraulic Head

1. Hydraulic head = total mechanical energy operating on a fluid mass per unit weight

   a. h = hydraulic head, measured in terms of length (*see below)

III. Hydraulic Head

A. Piezometer- device used to measure the total energy of fluid flowing through a pipe packed with sand.

1. open conduit at top and bottom
2. water rises into conduit in direction proportion to total fluid energy
3. Constructed of small-diameter well which measures hydraulic head at a point in the aquifer

B. Derivation of Hydraulic Head from Bernoulli Equation

$E_{tm} = (V^2/2) + gz + (P/p) = h$ (hydraulic head)

Since ground water velocity is so slow compared to the other components (e.g. avg = 10^{-8} m/s = 30 m/yr), the velocity portion of the equation essentially = 0... hence

$h = gz + (P/p) = z + (P/pg)$ by rearranging and substitution of $P = pgh_p$

** $h = z + h_p$ (units of measurement in L) **

here h = total hydraulic head, z = elevation of a reference point in water column above datum, h_p = point water pressure head = height of water column above reference point.

** $h_p = P/(pg)$ ** Rearranging **** $P = pgh_p$

where h_p = point water pressure head, P = fluid pressure, p = fluid density, g = gravity

(Rem: g = 9.8 m per sec^2; density of pure water = 1 gm/cm^3 = 1000 kg/m^3)

C. Derivation of Vertical Hydraulic Gradient

1. grad $h = (h_2 - h_1)/(L_2 - L_1)$

where grad h = hydraulic gradient (dimensionless ratio), h_2 = total head at point 2, h_1 = total head at point 1, $(L_2 - L_1)$ = horizontal distance between point 1 and 2.

Give Example Problem

IV. Force Potential and Hydraulic Head

A. Force Potential

1. Defined: total mechanical energy per unit mass of water
   a. sum: kinetic + elevation energy + pressure

   F.P. = gh

where F.P. = force potential, g = gravity = 9.8 m/sec. sq., h = total hydraulic head

** force potential is driving force behind ground water flow **

Remember $h = z + h_p$

where z = elevation of point above a datum, h_p = pressure head, height of column of water above reference point

** since g is constant, F.P. is controlled by total hydraulic head

Refer to examples in Figure 5.5

2. Ground Water Flow
   a. Always directed from high force potential to low force potential

      (1) largely controlled by total hydraulic head (measured in L units)

   b. Potentiometric Surface: contouring of force potential
      (1) Ground Water Contour Maps

V. Darcy's Law

A. Remember Darcy's Law ...

$$Q = KIA$$

Where Q = discharge (L^3/T)
K = hydraulic conductivity (permeability) (L/T)
I = hydraulic gradient (vertical head distance between two points of observation) (decimal ratio)
A = cross-sectional area through which flow occurs (L^2)

** This base equation was originally designed to solve for one dimensional flow through a sand filled pipe... Equation must be modified to solve for ground water flow through a porous media in three dimensions **

B. Applicability of Darcy's Law

1. Laminar vs. Turbulent Flow

a. Laminar Flow: fluid flow in which shear surfaces conform to the shape of the boundary of the fluid, molecules of water follow smooth line paths

(1) Laminar Flow Regime: at low shear rates, with relatively high resistance to shear

(2) resistance to shear > with > viscosity, more viscous fluids will tend towards laminar flow

b. Turbulent Flow: fluid flow is characterized by vortices and eddies,

(1) Characterized by higher rates of shear relative to viscosity of fluid

c. Reynolds Number: Analytical technique defining the conditions of laminar vs. turbulent flow

Defined by: $Re = \frac{pdv}{u} = \frac{\text{driving force}}{\text{resisting friction}}$ where :

p = fluid density, u = viscosity of fluid, d = diameter of passage way through which water moves, v = velocity of flowing medium, Re = dimensionless number defining laminar vs. turbulent flow.

** For pipes and open channels, transition from laminar to turbulent flow: Re = 500-2000.

2. Darcy's Law and Flow Regime (limiting factor)

a. Darcy's Law only valid under conditions of laminar ground water flow (Re < 10 based on experimentation with porous media)

(1) Under most conditions, ground water flow velocity is slow enough that Re is below critical value... i.e. Laminar Flow Conditions Prevail!

(2) Exceptions:
(a) Highly open conduits (karst terrane, vesicular basalt)
(b) Around high-discharge pumping well with exaggerated cone of depression.

C. Specific Discharge and Average Linear Velocity

1. Specific Discharge
a. Discharge Defined (water flow through open conduit)

$Q = vA$

Where Q = discharge (L^3/T), v = velocity (L/T), and A = area (L^2)

Rearrange equation to $v = Q/A$

From Darcy's Law $Q/A = -K(dh/dl)$

Via Substitution **** $v = -K(dh/dl)$ = Darcy's Flux

** where v = specific discharge = velocity ground water would move through an aquifer if it were an ideal open, pipe-like conduit (which it is not)

b. Aquifer Conduit

(1) Flow openings/conduits of an aquifer comprise a much smaller cross-sectional area than the dimensions of the aquifer itself.

(2) Effective conduit area for a porous media

$Ae = n_e A$

where Ae = effective area of porous media for flow, n_e = effective porosity of media, and A = total cross-sectional area of media.

2. Seepage Velocity or Average Linear Velocity

a. Modification of specific discharge (modifying Darcy's Law to account for effective pore-throat area)

**** $V_s = Q/(n_e A) = \frac{K\,dh}{n_e\,dl}$

where Vs = seepage velocity = average linear velocity of ground water molecule moving through actual pore throat openings of aquifer

b. Seepage velocity does not account for dispersion of molecules and solutes owing to complex flow paths through pore conduits

(1) Dispersion of dissolved chemical species

(a) Mechanical dispersion

(b) Chemical dispersion/diffusion

** Net result of dispersion, advective rate of ground water seepage velocity greater that rate of solute transport with ground water (i.e. the ground water may flow faster than chemical contamination it may be transporting!! More on this later...)

VI. Gradient of Hydraulic Head

A. Force Potential

1. Defined: total mechanical energy per unit mass of water

a. sum: kinetic + elevation energy + pressure

F.P. = gh

where F.P. = force potential, g = gravity = 9.8 m/sec. sq., h = total hydraulic head

E.g. As measured in a piezometer, a point in an aquifer has a head of 15.1 m and the value of g = 9.81 m/sec^2; then F.P. = 15.1 x 9.81 = 148.1 m sq./sec^2

** force potential is driving force behind ground water flow **

Remember $h = z + h_p$

where z = elevation of point above a datum, h_p = pressure head, height of column of water above reference point

** since g is constant, F.P. is controlled by total hydraulic head

B. Since g is constant, h is the variable of concern in determining ground water flow

1. Variable h throughout aquifer

a. ground water contour map: showing lines of equal head

(1) equipotential lines

2. Based on $h = z + h_p$, at any given point in an aquifer, equipotential lines are vertical throughout the aquifer

   a. Vertical equipotential lines intersect with the map surface to form contours

   b. Gradient of head (grad h)
      (1) Grad h is vector with direction and magnitude

grad h = dh/ds

grad h = gradient of head, dh = change in head over distance s, ds = change in distance

(a) Grad h has vector perpendicular to lines of equipotential

   c. Grad h must be positive for ground water to flow
      (1) Grad h = 0 = flat water table = no flow condition

VII. Relationship of Ground-Water Flow to Direction of Hydraulic Head

A. Controls of absolute ground water flow directions
   1. Grad h
   2. Degree of Anisotropy of Aquifer
      a. Anisotropy of hydraulic conductivity in 3-dimensions
      b. Orientation of axes of K with respect to grad h

B. Homogeneous, isotropic aquifers (K same in all directions)
   1. Ground water flow parallel to grad h
   2. Flow perpendicular to lines of equipotential

C. Anisotropic aquifer media
   1. Direction of ground water flow will be skewed from parallel to grad h (refer to attached Fig. 5.10)

VIII. Flow Lines and Flow Nets

A. Flow Line:
   1. Imaginary line that traces the path that a particle of ground water would follow as it flows through an aquifer

      a. Flow lines allow visualization of ground water movement

   2. Isotropic, homogeneous aquifers
      a. Flow lines drawn perpendicular to equipotential lines
      b. Flow lines parallel to grad h

   3. Anistropic aquifers (unequal K in plane of flow)
      a. flow line cross equipotential lines at an angle dictated by the degree of anistropy relative to grad h (refer to attached Fig. 5.10)

B. Flow Nets

1. Network of equipotential lines and flow lines

C. Construction of Flow Nets (Simple Case Scenario)

1. Base Assumptions
    a. Aquifer is homogeneous with respect to composition
    b. Aquifer is fully saturated with water (all pores filled)
    c. Aquifer is isotropic with respect to hydraulic properties
    d. The potential field (head conditions) is "steady state" over time
    e. The soil and water are incompressible
    f. Flow is laminar, Reynolds No. low, Darcy's Law is valid
    g. All boundary Conditions are known

2. Boundary Conditions
    a. No Flow Boundary
        (1) Ground water can not pass/penetrate a no flow boundary
            (a) Flow lines will be parallel to a no flow boundary
            (b) Equipotential lines will intersect the boundary at right angles

    b. Constant Head Boundary
        (1) Ground water may pass through a constant head boundary, head is the same everywhere along the constant head boundary.

            (a) Boundary = equipotential line
            (b) Adjacent equipotentials = parallel
            (c) Flow lines perpendicular to boundary

    c. Water Table Boundary (Unconfined aquifers only)
        (1) boundary represents a line where head is known
        (2) Recharge/discharge conditions across boundary
            (a) Flow lines at oblique angle to boundary

        (3) No recharge/discharge across boundary
            (a) Flow lines parallel to boundary

3. Steps for constructing a simplified flow net
    a. Flow net = family of equipotential lines with flow lines drawn orthogonal to them so that a pattern of "square-mesh" figures results.
        (1) Identify boundary conditions (see above)
        (2) Make sketch of boundaries to scale
        (3) Identify position of known equipotential and flow line conditions
        (4) Draw trial set of flow lines

            (a) Outer flow lines parallel to no flow boundary
            (b) Flow lines perpendicular to constant head boundary
            (c) Draw a set of flow lines at arbitrary equal distance from one another

(5) Draw a trial set of equipotential lines

(a) Equipotential lines perpendicular to flow lines

i) parallel to constant head boundary
ii) perpendicular to no flow boundary
iii) Draw equipotential lines so as to form equidimensional squares or square-like forms

IX. Steady Flow in a Confined Aquifer

A. Given steady flow in a confined aquifer, there will be a gradient or slope to the potentiometric surface of the aquifer

1. Ground water flow will be from high head to low head along the potentiometric surface in 2-dimensions

2. To determine the quantity of flow, Darcy's law may be used directly

$$Q = KAI = K(bw)I$$

where K = hydraulic conductivity, A = cross-sectional area (aquifer thickness (b) x aquifer width (w)), and I = grad h

3. To determine the quantity of flow "per unit width", assume w = 1

**** $Q = KbI$ (L^2/T)

X. Steady Flow in an Unconfined Aquifer

A. Water table aquifer by definition, water table serves as the upper boundary for ground water flow

1. Assuming no recharge to system, to maintain constant Q throughout the aquifer with head change over distance L, the gradient of the water table must increase in the direction of flow.

B. Dupuit Assumptions

1. hydraulic gradient = slope of water table
2. for small water table gradients, streamlines (i.e. flow lines) are horizontal and equipotential lines are vertical.

C. Dupuit Equation

**** $q' = 0.5K\,(h_1^2 - h_2^2)/L$

where q' = flow per unit width (L^2/T), K = hydraulic conductivity (L/T), I = gradient of water table between two points