PRINCIPLES OF GROUNDWATER FLOW
EQUATION SET

②

(1) $E_K = \frac{1}{2} m v^2$

E_K = KINETIC ENERGY
M = MASS
V = VELOCITY

(UNITS $\frac{Kg\ m^2}{Sec^2}$ = N-m)

(2) $E_g = m g z$

E_g = gravitational potential energy
m = MASS
g = gravity 9.81 m/sec²
z = vertical distance above datum

SAME

(3) $P = F/A$

P = pressure (M/LT^2)
f = force (ML/T^2)
A = AREA (L^2)

(4) $E_{TV} = \frac{1}{2} \rho v^2 + \rho g z + P$

E_{TV} = TOTAL ENERGY OF FLUID
ρ = density = m/v
v = velocity
g = gravity
z = distance/vertical
P = FLUID PRESSURE

(5) ★ BERNOULLI EQUATION

TOTAL ENERGY PER UNIT MASS

$$E_{Tm} = \frac{V^2}{2} + gz + \frac{P}{\rho}$$

[UNITS L^2/T^2]

(6) ✱ $E_{TW} = \frac{E_{TM}}{g}$

E_{TW} = TOTAL ENERGY PER UNIT WEIGHT

E_{TM} = TOTAL ENERGY PER UNIT MASS

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(7) HEAD EQUATION

$$h = z + \frac{P}{\rho g} \quad ★\ P = \rho g h_p$$

$$★\ h = z + h_p.$$

h = TOTAL HEAD

z = vertical ~~elevation~~ ELEVATION

h_p = POINT-WATER PRESSURE HEAD

---

(8) FORCE POTENTIAL

$$\Phi = gh$$

Φ = FORCE POTENTIAL (DRIVING FORCE)

g = gravity

h = TOTAL HEAD

---

(9) DARCY'S LAW

$$Q = -KA\ dh/dl$$

FOR LAMINAR FLOWS ONLY

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(10) REYNOLDS No.

$$R = \frac{\rho v d}{\mu}$$

R = REYNOLDS NO.

ρ = FLUID DENSITY

v = VELOCITY

d = diameter opening

μ = VISCOSITY

R = 1-10 = LAMINAR FLOW

R = 60-600 = TURBULENT FLOW

} GROUND H_2O

(11) · DARCY FLUX / SPECIFIC DISCHARGE

$Q = V/A$ Q = DISCHARGE A = AREA (L^3/T)
V = VELOCITY

$V = Q/A$

DARCY EQUATION $Q = -KIA$ $\frac{Q}{A} = -KI = V$

DARCY FLUX

(12) SEEPAGE VELOCITY

$$V_x = \frac{Q}{n_e A} = -\frac{KI}{n_e}$$ (UNITS L/T)

Q = DISCHARGE, n_e = effective porosity, A = AREA
V_x = SEEPAGE VELOCITY

(13) HYDRAULIC GRADIENT

$$\text{grad } h = \frac{dh}{ds} = \frac{\Delta \text{HEAD}}{\Delta \text{DISTANCE}}$$

(14) FLOW IN CONFINED AQUIFER DISCHARGE PER UNIT WIDTH

$$q' = Kb \frac{dh}{dl}$$ [UNITS L^2/T]

q' = discharge, b = AQUIFER THICKNESS, K = hydraulic CONDUCTI.
dh/dl = grad h

15) FLOW IN UNCONFINED AQUIFER (DUPUIT EQUATION)

$$q' = \frac{1}{2} K \left(\frac{h_1^2 - h_2^2}{L} \right)$$

E_k	Kinetic energy
E_{tm}	Total energy per unit mass
E_{tv}	Total energy per unit volume
f	Number of squares along a flowtube in a flow net
F	Force
g	Acceleration of gravity
grad h	Vector representing the gradient of head
h	Head
h_f	Fresh-water pressure head
h_p	Point-water pressure head (height of point fluid in piezometer)
K	Hydraulic conductivity
K′	Vertical hydraulic conductivity of an aquitard
L	Distance between h_1 and h_2 in the Dupuit equations
m	Mass
M	Mass of water in the derivation of the ground-water-flow equation
n	Porosity
n_e	Effective porosity
p	Number of flowtubes in a flow net
P	Pressure
P_0	Atmospheric pressure
Q	Discharge
q′	Flow per unit width
q'_x	Flow per unit width at location x
q_x	Discharge in the x-direction
q_y	Discharge in the y-direction
q_z	Discharge in the z-direction
R	Reynolds number
S	Storage coefficient
S_y	Specific yield
T	Transmissivity
v	Velocity
V_x	Average linear velocity
V_s	Volume of solids
w	Recharge rate to an unconfined aquifer
W	Work
x	Distance
z	Elevation
α	Compressibility of aquifer
β	Compressibility of water
Φ	Force potential
ρ	Fluid density
ρ_f	Density of fresh water
ρ_p	Density of point water
ρ_w	Density of water
μ	Viscosity
σ	Angle of refraction

Table A1.3 Conversion Factors FPS (foot-pound-second) System of Units to SI Units

	Multiply	By	To obtain
Length	ft	3.048×10^{-1}	m
	ft	3.048×10	cm
	ft	3.048×10^{-4}	km
	mile	1.609×10^{3}	m
	mile	1.609	km
Area	ft^2	9.290×10^{-2}	m^2
	mi^2	2.590	km^2
	acre	4.047×10^{3}	m^2
	acre	4.047×10^{-3}	km^2
Volume	ft^3	2.832×10^{-2}	m^3
	U.S. gal	3.785×10^{-3}	m^3
	U.K. gal	4.546×10^{-3}	m^3
	ft^3	2.832×10	ℓ
	U.S. gal	3.785	ℓ
	U.K. gal	4.546	ℓ
Velocity	ft/s	3.048×10^{-1}	m/s
	ft/s	3.048×10	cm/s
	mi/h	4.470×10^{-1}	m/s
	mi/h	1.609	km/h
Acceleration	ft/s^2	3.048×10^{-1}	m/s^2
Mass	lb_m*	4.536×10^{-1}	kg
	slug*	1.459×10	kg
	ton	1.016×10^{3}	kg
Force and weight	lb_f*	4.448	N
	poundal	1.383×10^{-1}	N
Pressure and stress	psi	6.895×10^{3}	Pa or N/m^2
	lb_f/ft^2	4.788×10	Pa
	poundal/ft^2	1.488	Pa
	atm	1.013×10^{5}	Pa
	in Hg	3.386×10^{3}	Pa
	mb	1.000×10^{2}	Pa
Work and energy	ft-lb_f	1.356	J
	ft-poundal	4.214×10^{-2}	J
	Btu	1.055×10^{-3}	J
	calorie	4.187	J
Mass density	lb/ft^3	1.602×10	kg/m^3
	slug/ft^3	5.154×10^{2}	kg/m^3
Weight density	lb_f/ft^3	1.571×10^{2}	N/m^3
Discharge	ft^3/s	2.832×10^{-2}	m^3/s
	ft^3/s	2.832×10	ℓ/s
	U.S. gal/min	6.309×10^{-5}	m^3/s
	U.K. gal/min	7.576×10^{-5}	m^3/s
	U.S. gal/min	6.309×10^{-2}	ℓ/s
	U.K. gal/min	7.576×10^{-2}	ℓ/s
Hydraulic conductivity	ft/s	3.048×10^{-1}	m/s
(see also Table 2.3.)	U.S. gal/day/ft^2	4.720×10^{-7}	m/s
Transmissivity	ft^2/s	9.290×10^{-2}	m^2/s
	U.S. gal/day/ft	1.438×10^{-7}	m^2/s

*A body whose mass is 1 lb mass (lb_m) has a weight of 1 lb force (lb_f). 1 lb_f is the force required to accelerate a body of 1 lb_m to an acceleration of $g = 32.2\ ft/s^2$. A slug is the unit of mass which, when acted upon by a force of 1 lb_f, acquires an acceleration of 1 ft/s^2.

Table 2.1 Dimensions and Common Units for Basic Groundwater Parameters*

Parameter	Symbol	Système International† SI		Foot-pound-second system,‡ FPS	
		Dimension	Units	Dimension	Units
Hydraulic head	h	$[L]$	m	$[L]$	ft
Pressure head	ψ	$[L]$	m	$[L]$	ft
Elevation head	z	$[L]$	m	$[L]$	ft
Fluid pressure	p	$[M/LT^2]$	N/m^2 or Pa	$[F/L^2]$	lb/ft^2
Fluid potential	Φ	$[L^2/T^2]$	m^2/s^2	$[L^2/T^2]$	ft^2/s^2
Mass density	ρ	$[M/L^3]$	kg/m^3	—	—
Weight density	γ	—	—	$[F/L^3]$	lb/ft^3
Specific discharge	v	$[L/T]$	m/s	$[L/T]$	ft/s
Hydraulic conductivity	K	$[L/T]$	m/s	$[L/T]$	ft/s

*See also Tables A1.1, A1.2, and A1.3, Appendix I.

†Basic dimensions are length $[L]$, mass $[M]$, and time $[T]$.

‡Basic dimensions are length $[L]$, force $[F]$, and time $[T]$.

Table A1.1 Definitions, Dimensions, and SI Units for Basic Mechanical Properties

Property	Symbol	Definition	SI unit	SI symbol	Dimension of unit: Derived	Dimension of unit: Basic
Mass	M		kilogram	kg		kg
Length	l		meter	m		m
Time	t		second	s		s
Area	A	$A = l^2$				m^2
Volume	V	$V = l^3$				m^3
Velocity	v	$v = l/t$				m/s
Acceleration	a	$a = l/t^2$				m/s^2
Force	F	$F = Ma$	newton	N	N	$kg \cdot m/s^2$
Weight	w	$w = Mg$	newton	N	N	$kg \cdot m/s^2$
Pressure	p	$p = F/A$	pascal	Pa	N/m^2	$kg/m \cdot s^2$
Work	W	$W = Fl$	joule	J	N·m	$kg \cdot m^2/s^2$
Energy		Work done	joule	J	N·m	$kg \cdot m^2/s^2$
Mass density	ρ	$\rho = M/V$				kg/m^3
Weight density	γ	$\gamma = w/V$			N/m^3	$kg/m^2 \cdot s^2$
Stress	σ, τ	Internal response to external p	pascal	Pa	N/m^2	$kg/m \cdot s^2$
Strain	ϵ	$\epsilon = \Delta V/V$				Dimensionless
Young's modulus	E	Hooke's law			N/m^2	$kg/m \cdot s^2$

Table A1.2 Definitions, Dimensions, and SI Units for Fluid Properties and Groundwater Terms

Property	Symbol	Definition	SI unit	SI symbol	Dimensions of unit: Derived	Dimensions of unit: Basic
Volume	V	$V = l^3$	liter (= $m^3 \times 10^{-3}$)	ℓ	ℓ	m^3
Discharge	Q	$Q = l^3/t$			ℓ/s	m^3/s
Fluid pressure	p	$p = F/A$	pascal	Pa	N/m^2	$kg/m \cdot s^2$
Head	h					m
Mass density	ρ	$\rho = M/V$				kg/m^3
Dynamic viscosity	μ	Newton's law	centipoise (= $N \cdot s/m^2 \times 10^{-3}$)	cP	cP, $N \cdot s/m^2$	$kg/m \cdot s$
Kinematic viscosity	ν	$\nu = \mu/\rho$	centistoke (= $m^2/s \times 10^{-6}$)	cSt	cSt	m^2/s
Compressibility	α, β	$\alpha = 1/E$			m^2/N	$m \cdot s^2/kg$
Hydraulic conductivity	K	Darcy's law			cm/s	m/s
Permeability	k	$k = K\mu/\rho g$			cm^2	m^2
Porosity	n					Dimensionless
Specific storage	S_s	$S_s = \rho g(\alpha + n\beta)$				1/m
Storativity	S	$S = S_s b$*				Dimensionless
Transmissivity	T	$T = Kb$*				m^2/s

*b, thickness of confined aquifer (see Section 2.10).