FINAL EXAM REVIEW SELECTED ANSWERS

This is a list of review problems provided to all MTH 111 students.

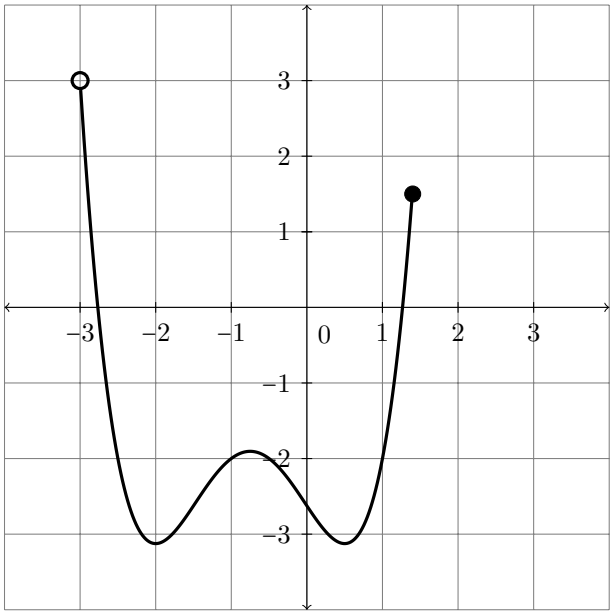
Be aware that not all topics are covered by this review.

Studying only these problems will NOT be enough preparation for the final exam. Please be sure to review all course materials in preparation for the final exam.

For the final exam, you may use

- a calculator (TI-84 PLUS CE or less)
- ONE 3×5 notecard of HANDWRITTEN notes to be turned in with your final exam.

1. Find all extrema. Be sure to specify what type of extrema it is.



local max: ≈ -1.9

Absolute min: ≈ -3.1

2. Sketch the graph of a polynomial with no absolute maximum but an absolute minimum of 2 at $x = -2$ and a local maximum of 6 at $x = 1$.

3. Describe the end behavior of a degree three polynomial with a leading coefficient of -257.

   As $x \to -\infty$, $y \to \infty$

   As $x \to \infty$, $y \to -\infty$

4. Use the Division Algorithm to write $f(x)$ in the form $f(x) = d(x) \cdot q(x) + r(x)$ where

$$f(x) = 3x^4 - 16x^3 - 19x^2 + 44x - 12 \qquad \text{and} \qquad d(x) = x^2 + x - 1$$

$3x^4 - 16x^3 - 19x^2 + 44x - 12 = (x^2 + x - 1)(3x^2 - 19x + 3) + 22x - 9$

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5. Solve

$$2x^5 + x^4 = -6x^3 - 3x^2$$

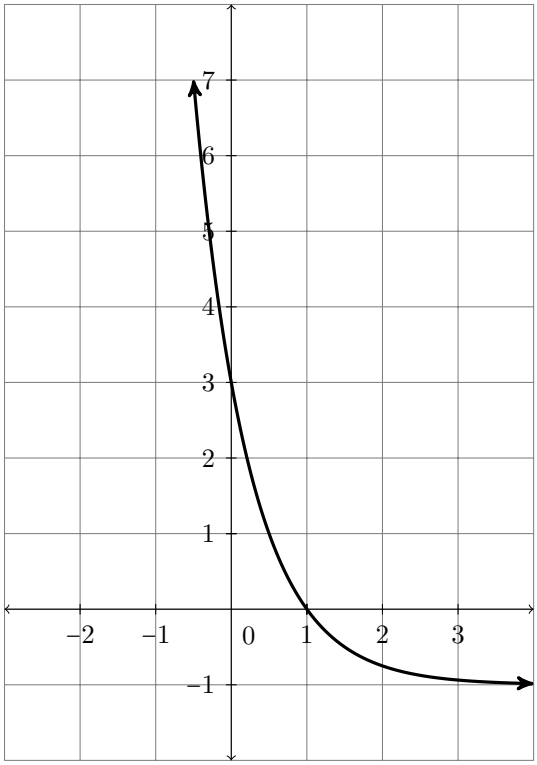
$x = -\frac{1}{2}, 0, \pm i\sqrt{3}$

6. Solve

$$2u^{-2} - 5u^{-1} = 3$$

$u = \frac{1}{3}, -2$

7. Using the graph of $g(x)$ below, find the average rate of change of g on [0,1].



The rate of change of g on [0,1] is 3.

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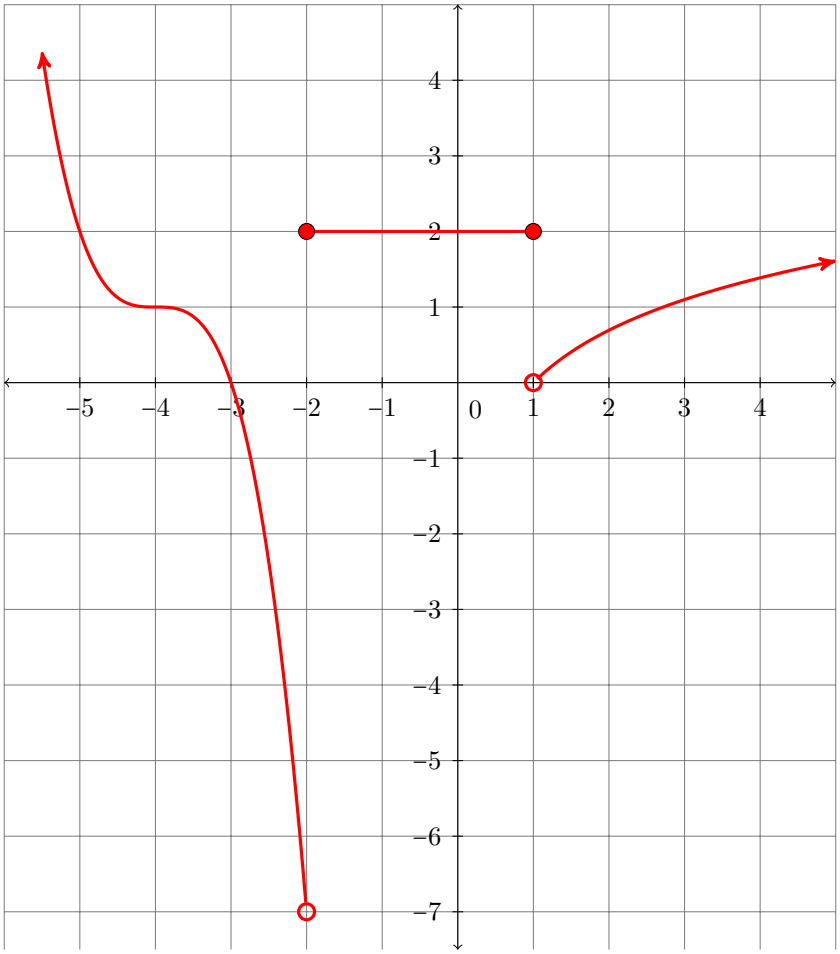
8. For the following, consider $f(x)$ given below.

$$f(x) = \begin{cases} -(x+4)^3 + 1 & \text{if } x < -2 \\ 2 & \text{if } -2 \le x \le 1 \\ \ln(x) & \text{if } x > 1 \end{cases}$$

(a) Find x such that $f(x) = 0$

$x = -3$

(b) Graph $f(x)$



9. The maximum load that a horizontal beam can carry is directly proportional to its width. If a beam 1.5 inches wide can support a load of 250 pounds, find the load that a beam of the same type can support if its width is 3.5 inches.

It can support a load of approximately 583.3 pounds.

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10. Simplify the following.

    (a) $6^{\log_6(x^2-1)}$

    $x^2 - 1$

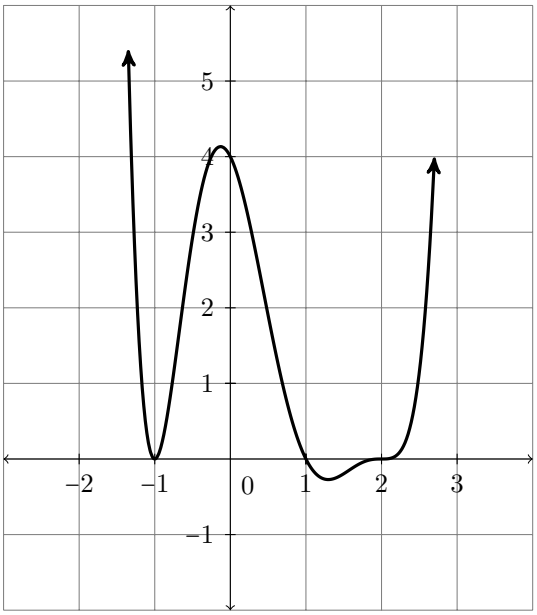
    (b) $\ln(e^x)$

    x

    (c) $\log_5 125$

    3

11. The graph of a polynomial $f(x)$ is given below.



    (a) Using the graph and smallest degree possible, write the complete factored form of f. *Hint: the leading coefficient is not ± 1*

    $f(x) = \frac{1}{2}(x+1)^2(x-1)(x-2)^3$

    (b) Solve $f(x) > 0$. Write your answer in interval notation.

    $(-\infty, -1) \cup (-1, 1) \cup (2, \infty)$

12. If a zero of k is $\frac{2}{3}$, find the complete factored form of $k(x) = 3x^4 - 5x^3 + 50x^2 - 80x + 32$

    $k(x) = (3x-2)(x-1)(x-4i)(x+4i)$

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13. For the following parts, consider the function, $h(x) = \dfrac{5x^2 + 18x - 8}{2x^2 + 5x - 12}$

    (a) State the domain of h.

    $\{x : x \neq -4, \dfrac{3}{2}\}$

    (b) State all vertical asymptotes (if any).

    $x = \dfrac{3}{2}$

    (c) State the horizontal asymptote.

    $y = \dfrac{5}{2}$

    (d) Does the graph cross its horizontal asymptote? Justify.

    No

14. How much money would you make if you invested \$8,200 in an account for 5 years that is compounded monthly at a rate of 6.8%? What about daily at a rate of 6%?

    You would make \$11,509.52 if monthly at 6.8% and \$11,068.57 if daily at 6%.

15. For the following, let $f(x) = (2x - 1)^5 + 7$.

    (a) Find f^{-1}.

    $f^{-1}(x) = \dfrac{1 + \sqrt[5]{x - 7}}{2}$

    (b) Verify that f and your answer to **15a** are inverses.

16. The tables for g and h are given below. Find the following. If it is not possible, then state so.

x	1	2	4	6	7	11
$g(x)$	-4	0	8	2	3	-2

x	0	2	4	6	8	11
$h(x)$	6	0	12	7	2	-3

(a) $(g+h)(2) = 0$ (b) $\left(\dfrac{h}{g}\right)(2)$ Not Possible (c) $(g \circ h)(0) = 2$

(d) $g^{-1}(2) = 6$ (e) $(g \cdot h)(6) = 14$ (f) $(g \circ h^{-1})(12) = 8$

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17. Solve
$$2\ln(x-1) = 14$$

$x = e^7 + 1$

18. Solve
$$3^{x-2} = 11^{7x}$$

$x = \dfrac{2\ln(3)}{\ln(3) - 7\ln(11)}$

19. Write as a single logarithm
$$\frac{1}{3}\log(2x) + 4\log z - 2\log y$$

$\log\left(\dfrac{z^4\sqrt[3]{2x}}{y^2}\right)$

20. Expand the logarithm. If possible, write your answer without exponents.
$$\ln\frac{y^4\sqrt{2x^2-1}}{z^3}$$

$4\ln(y) + \dfrac{1}{2}\ln(2x^2-1) - 3\ln(z)$

21. Solve
$$\log_2(2x) = 4 - \log_2(x+2)$$

$x = 2$

22. The voltage in a battery decreases by roughly 13% an hour when in use. Write a formula for $V(t)$ where $V(t)$ is the voltage of a 9 volt battery after t hours in use.

$V(t) = 9(.87)^t$

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23. Near New Guinea there is a relationship between the number of bird species found on an island and the size of the island. The table lists the number of species of birds y found on an island with an area of x square kilometers.

x (km^2)	0.1	1	10	100	1000
y (species)	10	15	20	25	30

(a) Use regression to find a function f that models the data.

$y = 15 + 2.17 \ln x$

(b) Predict the number of bird species on an island of 5,000 square kilometers. Did you use interpolation or extrapolation?

There will be about 33 bird species on an island of 5,000 square kilometers. I used extrapolation.

(c) If there are 50 bird species on an island, how many square kilometers would you expect the island to be?

It would be about 10,109,966 square kilometers.

24. Consider $h(x) = \log_4(-x + 1) - 3$

(a) State the domain of $h(x)$.

$(\infty, 1)$

(b) Graph $h(x)$. Be sure to include any intercepts or asymptotes.

(c) Briefly describe how you graphed h.

I moved the graph of $\log_4 x$
left 1
reflected over the y-axis
down 3

25. For the following, assume that you are given the graph of f. Describe how you would graph the function listed.

(a) $f(x) + 1$

(b) $4f(-x)$

(c) $\frac{1}{10}f(x - 3) - 20$

(d) $-3f(-x + 7) + 10$

(e) $8 - f(4x - 2)$

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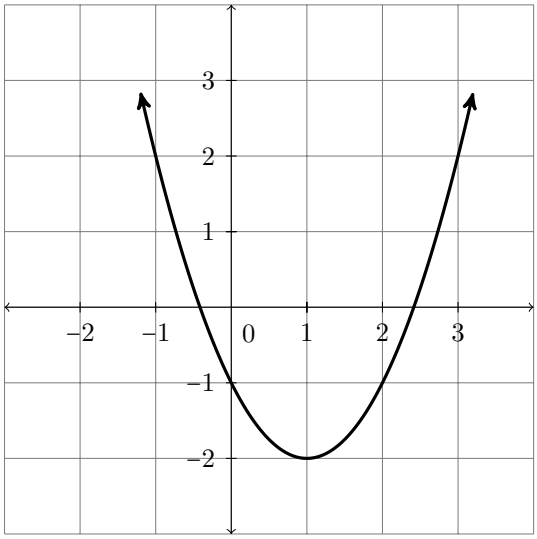
26. Using the graph below, determine if the function is even, odd, or neither. Justify your answer.



neither

27. For $f(x) = \sqrt[4]{x-5}$ and $g(x) = x^4 - 3$ find the functions AND state their domains.

(a) $(f+g)(x)$

$(f+g)(x) = x^4 - 3 + \sqrt[4]{x-5}$ $\quad$ $Domain: [5, \infty)$

(b) $(f \circ g)(x)$

$(f \circ g)(x) = x^4 - 3 + \sqrt[4]{x-5}$ $\quad$ $Domain: (-\infty, \sqrt[4]{8}\,] \cup [\sqrt[4]{8}, \infty)$

(c) $(g \circ f)(x)$

$(g \circ f)(x) = x - 8$ $\quad$ $Domain: [5, \infty)$