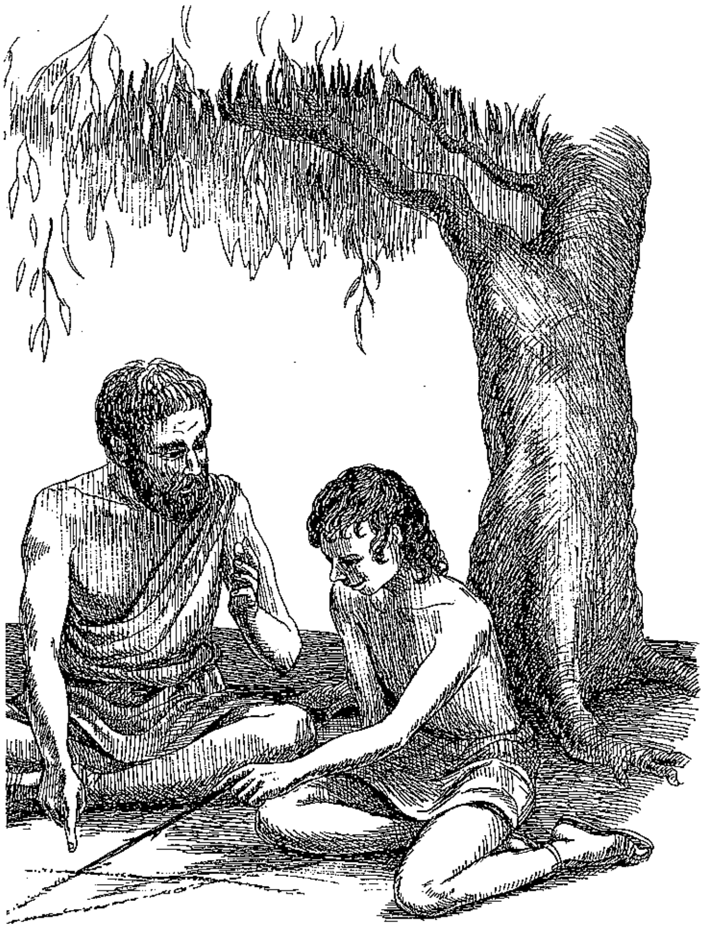


**PYTHAGORAS** *(pi-THAG-uh-rus)* **of Samos** (*c.* 560–*c.* 480 B.C.) was a Greek philosopher and religious leader, responsible for important developments in the history of mathematics, astronomy, and the theory of music. Pythagoras is most famous for the theorem on right triangles that bears his name.

# The Teacher Who Paid His Student

"Psst! Young man! Over here!"

The ragged Greek boy stopped in his tracks. Had he really heard someone calling him from behind that vegetable cart?

"Here! Here I am! Come here. I have an offer for you."

The boy, whose name was Philocrates, bent over to look around the wagon. The eyes he saw peering back at him looked a bit wild, but kind.

"What do you want with me?" answered Philocrates. "Surely you can see that I have no money to buy your wares! I'm just a poor street boy, trying to make a living doing odd jobs for anyone who will hire me."

"I have no wares to sell, except the truth," the stranger said. "Wouldn't you like to learn it?"

Philocrates scratched his head. He had met some unusual people, but this fellow seemed really different. The man's eyes parkled, and his manner seemed friendly enough. But truth? Iow could truth fill one's stomach?

"Sorry, friend," he replied. "I have to keep working the streets ) that my mother and sisters and I can eat each day. Perhaps you ın sell your truth to someone more wealthy than I."

He picked up his roughly woven sack of tools and waved a uick farewell.

"Wait! Please wait," the stranger called. "Let me introduce my-lf. My name is Pythagoras and I was born here on the island of ımos. But I have traveled to Miletus and Egypt and was even ptured and taken to Babylon for seven years. The things I have arned in these travels—oh, my son, you would be thrilled to arn them!"

"I'm sure I would, sir, but you don't understand my problem. I ıve no money, so I must work. It's that simple."

"All right," Pythagoras offered. "I'll make you a deal. If you will : me teach you, I will pay you what you would normally earn at ›ur other work." He paused to let his unusual proposition sink . "Well, what do you say? Shall we start tomorrow morning? ›u can meet me here by this bench."

Something drew Philocrates towards this odd teacher, but his actical nature made him resist. Finally he decided he would give ı try. If the stranger didn't really have any money to pay him for ıying student, he could always quit and go back to his odd jobs. hat did he have to lose?

"All right. We'll start tomorrow. But remember, I need daily ıges."

The next day the strange pair began their first lesson in the alley ıere they had met, amidst the cries of merchants and the min-





gled smells of fish, freshly baked honey cakes, and sweating donkeys carrying goods to sell. While the townspeople shopped and gossiped, Pythagoras and his student squatted in the dirt. The eager teacher drew shapes and figures on the ground. To Philocrates, it was all new but intriguing. And, just as he promised, at the end of the day Pythagoras paid.

Day after day it was the same. Each time Philocrates learned a new lesson Pythagoras paid him three oboli, about a penny. Soon he was making far more money than he could have made doing errands and odd jobs. He was an excellent student and quickly built up quite a savings account.

Pythagoras loved the arrangement, too. It was exhilarating to have an eager young mind absorb his ideas. Unfortunately, Philocrates learned so quickly and well that Pythagoras was soon out of money.

"I'm sorry to tell you this, Philocrates, but today will be our last lesson. I have no more money to pay your wages, so you will have to find other ways to support yourself."

"But Pythagoras, you can't quit teaching me now," the boy protested. "I'm just starting to understand arithmetic and you were going to teach me astronomy and geometry, too."

"I'm sorry, young man, but I see no other choice."

Philocrates hung his head and thought. In a moment he came up with an idea.

"I know! You have been paying me to learn; now I will pay you to teach."

So for the next several months the two continued to meet, but this time the student paid the teacher. By the time the lessons were completed, Pythagoras had become an experienced teacher, and

Philocrates had gained an excellent education!

Pythagoras's first "school" with Philocrates may have had only one student. But several years later he founded a real school at Croton, a Greek colony in southern Italy. This school became so influential it changed even the way people thought about knowledge. During his many travels, Pythagoras had gained quite a reputation. Some people even thought he was divine, or the son of their god Apollo. When he called together a group of wealthy scholars to form a school, no wonder many responded enthusiastically.

The students in Pythagoras's school were all adults. He divided them into two grades depending on their knowledge. The first grade was called the *acoustici*, or the listeners. They were invited to listen to Pythagoras lecture but were not allowed to see him—they had not yet proven themselves worthy. He stood behind a curtain, where only the second grade, the *mathematici*, could see him.

After three years of listening to their teacher's voice, the *acoustici* were admitted into the inner circle of learners. Seeing Pythagoras must have been worth waiting for. He had a flair for the dramatic and dressed like a stage performer. While the students waited for Pythagoras's entrance, musicians played popular music. Finally the curtain was drawn back and Pythagoras, stately in his white robe, appeared before the learners. His feet were strapped with gold sandals, and his head was crowned with a golden wreath. No wonder people suspected him of having gods for ancestors.

Pythagoras worked most of his problems in the sand. His classroom always had a good supply of sand on the floor, and his





attendants stood by with a selection of differently-colored sand in containers. When Pythagoras wanted to show one part of a geometric shape, for instance, the attendants would fill that part with blue or green sand so that students could see it more easily.

Pythagoras gave lectures on "mathemata," which in his language meant studies of all kinds. Because Pythagoras emphasized arithmetic and geometry, the word came to mean mathematics as we know it today. He also taught astronomy and music, but he believed that everything in the universe depended on numbers. Pythagoras and his followers chose the motto "All is Number." They were convinced that if they understood numbers, they would hold the key to life itself.

Because Pythagoras and his students believed that knowledge was powerful, they wanted to control it. They became secretive about what they knew. The school was a "Secret Brotherhood," and everyone who joined had to promise never to tell outsiders about their discoveries. If anyone did tell, the results could be disastrous for him or her.

"Have you heard about Hippasus?"

The question hummed throughout Croton.

"Yes. Isn't it horrible? Just because he broke the code of the Brotherhood. It doesn't seem fair."

"But the gods are always fair. He knew better than to tell about the discovery of irrational numbers."

"He must have known he would be expelled from Pythagoras's Secret Brotherhood. Do you suppose he thought that would be his only punishment?"

"I don't know. But there's something suspicious about the way he drowned, falling off that boat in such calm weather."

People were always talking about the Secret Brotherhood, also known as the Pythagorean School. Schools of adults were common, but this group had some unusual ideas. They became a kind of religious order with their own set of initiations and rites.

The 300 members of the Brotherhood shared whatever they had with each other. They were unusually kind to animals because they believed that human souls might come back after death for another life in an animal body. They were vegetarian and would not even wear wool because it came from sheep. If they could choose, they always took a low road instead of a high road, to show their humility. They would not poke a fire with iron because fire was the symbol of truth. They would not touch white roosters





or eat beans, because both roosters and beans symbolized perfection. On their clothing they each wore their sacred symbol—the pentagram, a five-pointed star.

In one way the Brotherhood was unusually progressive. During Pythagoras's day, women were forbidden to attend public meetings of any kind, but Pythagoras welcomed them to his school. Of course, they had to prove themselves just as the male students did. Nevertheless, at one time the select *mathematici* class included at least 28 women.

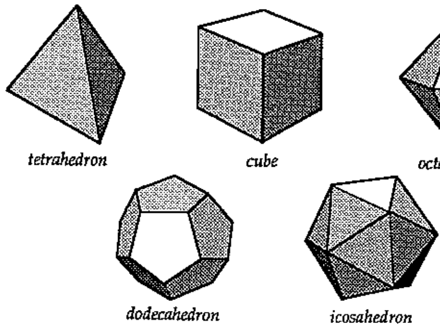
Because the Pythagoreans shared everything, it is hard to separate Pythagoras's discoveries from those of his followers. Much of modern mathematics is based on their work. Like Thales before him, Pythagoras insisted on mathematical proof. It was not enough to say that two angles were equal because they looked equal. One had to prove it. Pythagoras is most famous for providing the first logical proof of this theorem:

> *In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.*

The common formula for this theorem, if $c$ is the length of the hypotenuse and $a$ and $b$ the lengths of the other two sides, is

$$a^2 + b^2 = c^2.$$

The Pythagoreans were also the first to divide all numbers into even and odd. They learned to construct the five regular solids, the only solids whose faces are all the same shape and size: the tetrahedron (four sides), the cube (six sides), the octahedron (eight sides), the dodecahedron (twelve sides), and the icosahedron (twenty sides). The first two had been known from ancient times, but the others had never been constructed.




*The Pythagoreans learned to construct the five regular solids.*

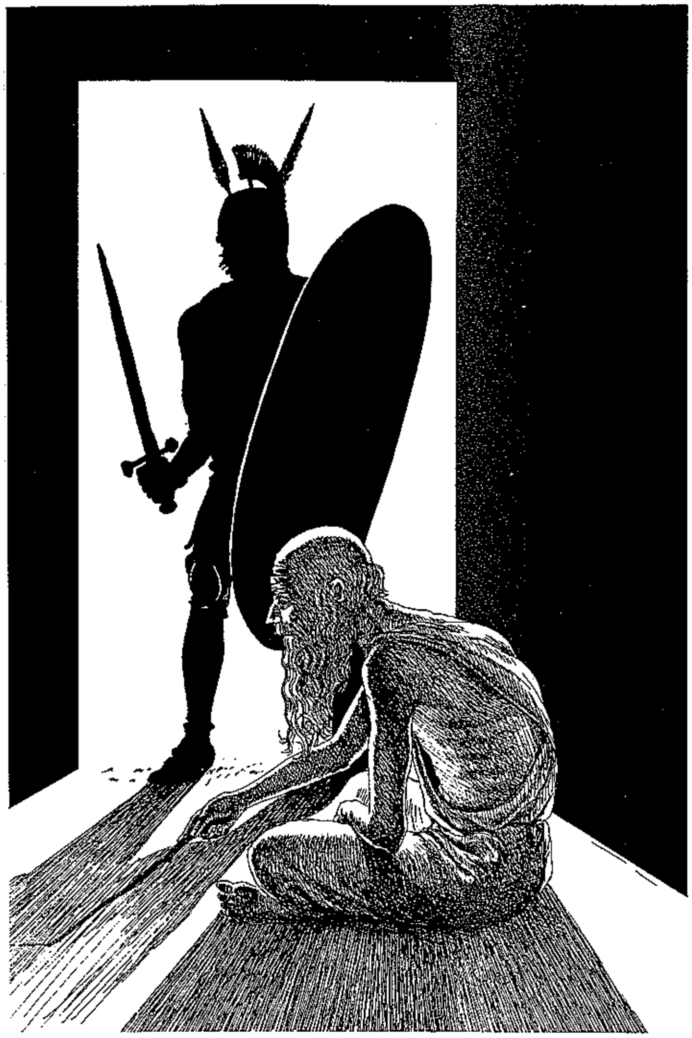
Great thinkers are not always appreciated in their own times. The Pythagoreans were often misunderstood. Many of their ideas and practices seemed strange to their countrymen. Some townspeople suspected the Pythagoreans would try to take over the local government. They blamed the Pythagoreans, who were quite wealthy, for trying to keep them poor. One day in about 500 B.C., an angry mob set the Pythagoreans' meetinghouse on fire during a lecture. Only a few members survived, and Pythagoras himself was killed. Some say that his students formed a human bridge over the fire so that he could escape—but when he reached a field of beans, he surrendered to his enemies rather than trample the sacred bean plants.

By this time chapters of the Brotherhood had spread throughout Sicily and southern Italy. For many years men and women continued to discuss the ideas Pythagoras had introduced. Today, all students of geometry and higher mathematics work with concepts





that Pythagoras discovered. The search for knowledge and truth continued long after Pythagoras's death and the end of the Brotherhood. It continues today wherever people are willing to pursue it.



**ARCHIMEDES** (*ar-ki-MEE-deez*), 287–212 B.C., was the greatest mathematician of ancient times. He made many original contributions to geometry and laid the foundation for integral calculus. His inventions include the catapult and Archimedes's screw.

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# The Man Who Concentrated Too Hard

Gleaming and glistening in the sun, the *Syracosia* was so bright the king and all his subjects had to squint to look at her. She sat in the shipyard, waiting to be launched. This was no ordinary ship, that was for certain. The *Syracosia* was crafted of the finest materials and, stern to bow, decked out with the latest in modern devices and luxurious furnishings.

There was just one little problem. In his desire to please and impress King Ptolemy of Alexandria, who had ordered the ship, King Hieron had built it too big. Now that it was finished, there was no way to get it into the water. King Hieron decided to use the tactic that had solved so many of his problems.

"Send for Archimedes!"

Archimedes was never thrilled about having his own work interrupted. However, he *had* boasted to the king once that if he were given someplace to stand he could move the whole earth. This is because Archimedes had discovered the law of the *lever*. By using a lever, people can easily lift objects much heavier than their own weight. It is this law that makes seesaws work. Archimedes's levers could move very heavy objects. Now Hieron wanted him to prove he could move a big ship, and Archimedes looked forward to the challenge.

When he got to the dry dock where the ship had been built, Archimedes was amazed at the crowds gathered there. Columns of workers loaded the ship with cargo. On another ramp, stewards helped passengers onto the decks. Hieron was determined to make the ship as heavy as possible. If he turned the problem into a test of Archimedes's creativity, perhaps he could cover his embarrassment at having built such a monster!

Archimedes went to work. With the help of some young boys who worked for him, he placed a collection of pulleys, levers, and ropes around the base of the ship. Everyone knew that these mechanical devices could never move so great a ship as the *Syracosia*. What did Archimedes think he was going to do with those contraptions?

Finally, everything was ready. Archimedes went to sit in a low chair on the beach. At a signal from Hieron, Archimedes gently tugged on the rope attached to the first pulley. Smoothly, almost effortlessly, the beautiful ship began to move towards the sea. Soon the ship glided into the water, while the puzzled townspeople cheered and scratched their heads.

Archimedes was a great problem solver. He liked to think about scientific problems, such as how to measure the circumference of a circle. However, he was willing to help with the practical problems in his home town, Syracuse. At that time, Syracuse was a part of Greece. Archimedes had been born there as had his father





Pheidias, and he was fond of the city. Situated on the coast of the island of Sicily, Syracuse was rapidly becoming an important trade center. It was also a perfect spot for thinking. Archimedes had gone to Alexandria to study when he was a young man, but even the good friends he met there could not draw him away from Syracuse.

Archimedes's father, Pheidias, had been a noted astronomer. That could be why Archimedes found the sun, the moon, and the planets so fascinating. One day he started thinking about big numbers. Most people believed that the number indicating how many grains of sand filled the seashore didn't exist. The sand couldn't be counted, so the number must be too big to imagine. Archimedes loved a challenge; he set out to count the sand. As if that wasn't hard enough, he decided to count how much sand it would take to fill up the entire known universe. (In those days, people thought the universe was the space between the sun, moon, and the five planets identified at that time—Venus, Mercury, Mars, Jupiter, and Saturn.)

First, Archimedes counted how many grains of sand made up a cluster the size of a poppy seed. Then he counted how many poppy seeds equaled the size of a man's finger. He calculated how many fingers it would take to fill a stadium. Archimedes kept going like this until he came up with his answer: $10^{63}$ grains.* This number illustrates Archimedes's new system for writing large numbers. He used *exponents* to indicate how many times a number should be multiplied by itself.

Archimedes's estimate of the size of the universe was too small, but he did count how much sand it would take to fill that space.

**Read this number as "ten to the 63rd power." Sixty-three is an* exponent.

This introduced a new way of thinking about numbers. From then on, no number was too big.

"Archimedes," said King Hieron one day, "I have another problem for you."

"Yes, your Majesty. How can I help?"

"This situation," the king said carefully, "is a bit uncomfortable for me. Will you promise to tell no one of my dilemma?"

"Of course. You know you can trust me."

"Yes, yes. Here is my problem. I have had a new crown crafted by Dionthenes. It's here, wrapped in this velvet cloth."

"It's magnificent, your Highness. Aren't you happy with it?"

"I'm pleased with the way it looks, but something tells me that it isn't quite as I ordered," the king explained. "You see, I took the required amount of gold to Dionthenes's shop and asked him to craft this crown. But now I wonder if the goldsmith substituted silver inside the crown, and kept some of the more valuable gold for himself."

"Hmmm." Archimedes was stumped. "Naturally, you wouldn't want to cut into such a beautiful crown. I suppose you've checked it to see that it weighs the same as the gold you gave Dionthenes?"

King Hieron nodded. He looked a bit forlorn that Archimedes hadn't solved his problem instantly.

Archimedes promised to think about it and wandered off, absent-mindedly talking to himself. He often got so absorbed in problems that he forgot to watch where he was going, bumping into merchants and walking in front of carts. Sometimes he forgot about eating until his friends reminded him.

One day Archimedes was relaxing at the public bath house. As he sat in the tub, an idea struck like lightning. He was so excited





about his discovery that he jumped up and ran down the streets shouting "Eureka, eureka!" ("I have found it!"). Archimedes had discovered the natural law of *buoyancy*. However, most people would have remembered to dress before running down the street!

In the bath, Archimedes observed the amount of water displaced by his body. He noticed that the water level rose and fell as he lowered and lifted his weight, and that the water buoyed him up more when his body was completely submerged. The water's *buoyancy* kept him afloat. As he thought about it, he realized that the greater the volume for the water to support, the greater the buoyancy. Large objects—ones with greater volume—are more buoyant than smaller objects of the same weight. For example, a six-inch plastic ruler and a nickel weigh the same, but the ruler floats and the nickel sinks. The ruler has greater volume and is therefore more buoyant because it has more volume for the water to support.

Archimedes knew that the crown Dionthenes designed for King Hieron weighed the same as the gold that the king gave him. But if the crown displaced more water than the gold, that meant its *volume* must be greater, so it must be filled with something less dense than gold—like silver. He put the gold under water and measured how much the water rose. Then he did the same thing with the crown, and the water rose higher—proof that the crown had greater volume and could not be pure gold. The king's suspicions were confirmed. No one knows for sure what happened to the craftsman, but it's not pleasant to imagine.

When he wasn't solving problems for the king, Archimedes was busy at work on his own discoveries. He wrote many pamphlets about his findings. Some of these can be seen in museums today.

Archimedes often conducted studies of the circle. He was determined to find a precise ratio between the *circumference* of a circle and its *diameter*. To do this, he drew a many-sided *polygon* inside a circle and then drew another polygon enclosing the circle. This helped him come up with a remarkably accurate *ratio*. After much experimentation, Archimedes announced that it was less than $3\frac{1}{7}$ and greater than $3\frac{10}{71}$. This ratio, called *pi* ($\pi$), has now been calculated to many decimal places.

Archimedes's work on circles led him to study all sorts of curves, spheres, and spirals. He spent much time drawing and making models, carefully writing down his observations. If writing materials were not handy, he spread out cold ashes from the fire and drew in them. Often he worked in the sand. Finally he put together his proofs for finding areas, volumes, and centers of gravity for curves, surfaces, circles, spheres, conics, and spirals. The achievement that Archimedes was proudest of was his discovery of how to calculate the volume of a sphere. He found that it equals two-thirds the volume of the smallest cylinder that will enclose it. He was so proud of this discovery that he requested that the sphere-and-cylinder diagram he worked with be engraved on his tombstone. Archimedes made so much progress in his studies that further work in this area was not possible until new tools were developed eighteen centuries later.

"Have you ever considered how wealthy you could be if you worked on practical things instead of all these figures?" asked one of Archimedes's neighbors one day.

"Perhaps money and fame are important to some. I would rather work at what really interests me," Archimedes responded.





"But you invented the water screw, didn't you? That's practical, isn't it?"

"I suppose everyone must do some things to help others," Archimedes mused, "but I would feel like a failure if I spent my whole life designing mechanical toys."

Compared to his mathematical research, inventions like the water screw seemed like toys to Archimedes. But the water screw was actually an important and practical tool. It was a tube, open at each end, that enclosed a rod. When the lower end was immersed in water at an angle and the upper end was rotated, the screw worked like a straw, drawing the water up and out the top. In Egypt, farmers used this invention to get water out of the Nile River for irrigation. Seamen also used the screw to bail water out of their ships. In Spain the screw was used to pump water out of silver mines being excavated.

Towards the end of his life Archimedes *had* to give more of his time to such practical devices. Sicily, the island of which Syracuse was capital, found itself caught between Rome and Carthage, which were fighting the Punic Wars. After King Hieron died, his grandson Hieronymus took over Sicily and foolishly allied the nation with Carthage. The Romans planned to attack and easily conquer Sicily, but they had not planned on Archimedes's creative weaponry.

Under Archimedes's direction, the military began an aggressive building program. First he showed them how to set up huge catapults, machines that could throw heavy stones over the city walls onto ships in the harbor. Then they built long poles on bases that could be wheeled to any part of the wall. These projected over the wall, and when a lever was released, dropped ship-destroying weights on the enemy below. Archimedes also designed massive cranes that could reach over the wall, pick up a ship, and dump all the sailors on board into the sea.

The Romans tried for two years to capture Syracuse, but without success. After seeing so many of Archimedes's ingenious weapons, some of the Roman sailors were understandably nervous. Sometimes if a ship got too close to the harbor, the Greeks would just hang a loose rope over the wall. The Romans were sure it was another of Archimedes's inventions to destroy them, and would flee in fear for their lives.

The Roman commander Marcellus was frustrated and angry at being defeated by a mathematician. But he was patient. He waited for a day when the Greeks would let down their guard. It happened during a festival honoring Artemis, Greek goddess of the moon. The Syracuse soldiers relaxed and took a holiday, eating and drinking so much they forgot to keep a sharp lookout. Before they knew what had happened, the Romans had conquered the city.

Archimedes had angered Marcellus, but Marcellus respected the mathematician's brilliance and creativity. Before his soldiers left ship, Marcellus gave strict instructions that Archimedes should be captured alive: no harm should come to him.

Unlike his countrymen, Archimedes was not celebrating when the Romans attacked. As usual, he was deeply involved in a problem. He had no idea anything unusual was happening, but he knew someone was standing in the light, making a shadow on the ground where he was drawing his geometric shapes.

"Get out of the light, will you?" he said. "Can't you see I'm working?"





"Rise and follow me, old man!" the soldier commanded.

Archimedes didn't hear him. He kept working, his brow furrowed in deep concentration. He didn't notice the soldier angrily unsheath his sword. Proud and arrogant, the soldier disobeyed Marcellus's order and killed the greatest creative genius of the ancient world.

Marcellus was deeply grieved. He immediately planned an elaborate ceremony to honor Archimedes. He did everything he could to comfort Archimedes's family. And he made sure that, as Archimedes had requested years earlier, a picture of a sphere within a cylinder was engraved on his tombstone.

**LEONARD OF PISA** or **FIBONACCI** (fee-boh-NAH-chee), ca. 1180–1250, was an Italian mathematician who popularized the use of Hindu-Arabic numerals throughout Europe. He worked in algebra and geometry, and introduced the Fibonacci sequence.

# Lean on the Blockhead

It was never comfortable sitting cross-legged on the floor at school, but Leonard was especially restless today. He could hear the murmuring of the crowd through the open window.

"Leonard, what is your solution to this problem?" his teacher asked.

Leonard was straining to hear what was going on outside.

"Leonard, I am speaking to you!" his teacher threatened. "Why aren't you paying attention?"

Leonard looked down at his wax tablet. Where his answer to the problem should have been, the wax was smooth. His writing instrument, a bone stylus, was upside down on his lap. He didn't even remember what the problem was!

"I'm sorry, Sir," he stammered. "I suppose I was thinking about the tower again."

It wasn't the first time Leonard had been caught daydreaming. In fact, the excitement on the streets of Pisa was distracting all the boys. Who could concentrate on geometry when there was a *real* problem to solve just outside the window?

The beautiful cathedral in Pisa had recently been completed and work had begun on the bell tower. Marble blocks had been carefully fitted into place, layer upon layer, until the tower was now three stories tall. The engineers planned for eight stories, but something had obviously gone wrong. At first the people of Pisa hoped it was just an illusion, but their hopes were soon dashed. The tower was leaning.

Everyone, even people with no building experience, had an opinion. Sometimes, Leonard's head seemed to spin as people shouted suggestions in the street.

"If you ask me, I think we should take the whole thing down and start over."

"Are you crazy? We've worked more than a year to get this far. Starting over would take forever!"

"Someone must have measured the blocks incorrectly. Why can't we get decent workers nowadays?"

Finally, the cause of the problem was determined. Because the soil in Pisa was very sandy, the foundation had settled unevenly. There was no way to correct that now, but the engineer in charge of the project had a plan.

"We'll thicken the remaining layers of blocks on the leaning side," he announced. "By the time we reach the eighth story, the tower will no longer lean. It should be perfectly straight."

The engineer's plan was carried out, but it backfired. When the extra weight was added to the leaning side, it sank even more. By the time the tower was finished two centuries later, it leaned almost seventeen feet!

Leonard was fascinated by the tower of Pisa, and had begun to understand how important it was to plan carefully. He also





thought it was interesting that people had different ideas about how to solve problems. "Someday," he thought, "I hope I can be good at solving problems."

Life in Pisa was never dull, even without the tower dilemma. Leonard especially loved the life at the busy wharf, because every week ships arrived in Pisa from faraway places. Leonard's favorite pastime was to go down to the docks and watch the ships as they were loaded and unloaded. Each merchant had several employees to count cargo, and to keep careful records of receipts and expenditures. Leonard watched as their fingers flew over their abacuses. He marveled at how quickly they calculated on their boards of beads. They recorded their totals using Roman numerals, sometimes making a long row of figures in their books.

Even as a young boy, Leonard thought there must be an easier way to keep financial records. Roman numerals were okay for adding or subtracting, but impractical for multiplication and division. The abacus worked well, but it was clumsy. Besides, there was no way to check your work except to do it all over again. If you got a different answer the second time, you'd have to do it over, or get a friend to try on another abacus.

One day when Leonard was about twelve years old, his father, Bonacci, called him aside with some news.

"Leonard, you know that Pisa controls a port in Algeria, don't you?"

"Yes, Father. I think everyone knows about the customs house in Bougie."

His father paused, choosing his words carefully. "Bougie is a beautiful city, filled with people from many exotic places."

Leonard was puzzled and impatient. He wanted to get over to the pier where a ship was just docking. "What is it, Father? Why are you telling me this?"

"Leonard, I have been assigned to be the chief officer there, in Bougie, beginning immediately. It's quite a good promotion for me, and I hope it won't upset your life too much."

"What do you mean?"

"You see, I must go as soon as possible. In a short time, I will send for you. Until then, you must be strong and work hard in school."

Leonard was sorry to think of leaving Pisa and his friends. But he looked forward to the journey across the Mediterranean Sea. He had heard stories about the coast of North Africa, and he could hardly wait to see it for himself. He loved and admired his father a great deal, and he determined to make his father proud.

"Of course, Father," Leonard said bravely. "I will stay here and take care of things until you send for me. Promise me it will be soon!"

To show his loyalty to his father, Leonard chose a nickname for himself. "From now on," he announced to his friends at school, "my name will be Fibonacci, which means Son of Bonacci."

After joining his father in Algeria, young Fibonacci learned far more than he had in school. Bougie was a wealthy city, full of cultural richness and diversity. Scholars from all around the world came to share their ideas. Some were busy translating great Greek literature into Arabic. Others excitedly discussed the latest scientific discoveries.

Fibonacci stared wide-eyed at everything he saw. As he walked along the wharf, he admired the colorful costumes the sailors





wore. Some of them spoke in unusual languages, and Fibonacci imagined they were telling of dangerous days on the sea.

The merchants here were using a system of bookkeeping that intrigued Fibonacci. They made strange marks on their tablets that did not resemble Roman numerals at all. Once, as he stood staring at a tablet over the shoulder of an accountant, the man began to grumble.

"What's the idea, young man? Why don't you mind your own business?"

Fibonacci hurried on his way. But after awhile, he noticed a grandfatherly man also keeping accounts, and decided to investigate.

"Excuse me, sir. May I ask you a question?" he asked timidly.

"Forty-seven, forty-eight, forty-nine . . . what? Oh bother, where was I?"

Fibonacci was afraid he would be chased off like a common wharf rat, but he stood his ground.

"I'd like to know what those marks are that you're making in your book, sir," he explained.

"What? These?" the man asked, pointing to a row of numbers. "Why, these are numbers, son, Hindu numbers. Best way to keep accounts ever invented."

After just a short lesson, Fibonacci was on his way. He whistled as he walked down the pier, proud and excited about what he had learned. Wouldn't it be great if everyone knew about the Hindu numbers?

When Fibonacci was older, he set out on his own to travel. He spent several years in Constantinople, then visited Egypt, Syria, Sicily, and Provence. Everywhere he went, he looked for other people who also were interested in numbers. Sometimes he helped them with problems, and sometimes they had answers to questions he had puzzled over for years.

Finally Fibonacci settled down in Pisa to write. His book, *Liber Abaci* (Book of the Abacus), was published in 1202. (At that time there were no printing presses. Every page of every book had to be painstakingly copied by hand.) Fibonacci was eager to see how people would respond to his ideas, particularly since he knew that some of his suggestions might be considered radical.

Fibonacci's book began this way:

> "The nine Hindu figures are: 1, 2, 3, 4, 5, 6, 7, 8, 9. With these nine figures, and with the sign 0, any number may be written."

Fibonacci proposed that Italy, and all of Europe, should use Hindu-Arabic numerals instead of Roman numerals. He cited many examples to demonstrate how much easier it was to multiply and divide using digits. At first, most people refused to consider the plan. After all, they argued, Roman numerals had worked just fine for centuries!

Some of Fibonacci's other ideas also took some getting used to. Most of his readers were not accustomed to using zero as a placeholder. He introduced a new way of writing fractions, too, placing a bar between the numerator and the denominator. Of course, since Arabs read from right to left, the fraction was on the left side of the whole number.

Fibonacci introduced a famous number sequence in his book. In this sequence each number is the sum of the two numbers preceding it. Fibonacci's sequence was the first sequence known in Europe in which the relation between two or more successive





terms could be expressed by a formula. Here are the first terms in the sequence:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55

Mathematicians have been intrigued by this sequence ever since Fibonacci identified it. In the 1800s, many surprising applications of the Fibonacci sequence were discovered in nature. Botanists found that the patterns of leaf buds on many stems follow the sequence, as do the spirals of seeds in the heads of sunflowers, the petals on artichokes, and the scales on pineapples. The more one looks, the more one sees them!

As a young boy, Fibonacci had chosen his own nickname. When he was older, he chose another. Sometimes he called himself "Leonardo Bigollo." The word *bigollo* has more than one meaning. It may mean *traveler*, which Fibonacci certainly was, but it may also mean *blockhead*. When skeptics wanted to ridicule his ideas, they taunted Fibonacci by calling him the *Bigollo*.

Some people think that Fibonacci especially enjoyed using this nickname as his signature on his later work. It was rewarding to show the European world what a blockhead could do! No one would laugh at Fibonacci today, though, because he is considered the greatest mathematician of the Middle Ages.