Homework Set 1 SOLUTIONS

1. How many people must be chosen to be sure that at least five have the same last four digits of their cell phone number?

There are 10000 possible 4-digit extensions. The worse case scenario is if there are 4 people with each of the extensions. That would mean you could have 40,000 people and still not have 5 people with one of the extensions. If you add one more person, then you are guaranteed that (at least) 5 people have the same 4-digit extension.

2. (E.C.) Show that at a party of 14 people, there are 2 people who have same number of friends.

There are 14 possibilities for the number of friends a person can have {0,1, …13}. If a person has 13 friends, then she is friends with everyone there.

If each of the 14 people had a different number of friends, then that would mean someone (Bob) would have 0 friends and someone would have 13 friends (Alice). This can't be since Alice would be friends with Bob (who has zero friends).

Therefore, it must be that the # of friends each person has is either a subset of {0, 1, …, 12} or a subset of {1, 2, …, 13}. Either way, you have 13 #'s in the set and 14 people, so two people must have the same number of friends.

3. A bag contains seven yellow marbles, six red marbles, and eight black marbles. Without looking in the bag, Igor removes N marbles all at once. If he is to be sure, no matter which choice of N marbles he removes, there are at least three marbles of one color and at least four marbles of another color left in the bag, what is the maximum possible value of N?

The most number of marbles that can be in the bag and still NOT satisfy the requirement occurs when there are 8 black marbles, 2 red marbles and 2 yellow marbles still in the bag.

If I add one more marble (it will either be a red or yellow, since there are no more blacks left to add back into the bag), then we are guaranteed to satisfy the requirement of at least 4 of one color and at least 3 of another color. This would mean you had (8+2+2) + 1 = 13 marbles in the bag, so you took 8 marbles.

4. Given 11 French books, 19 Spanish books, 7 German books, 17 Russian books, and 25 Italian books, how many books must be chosen to guarantee there are 12 books of the same language?

The worst case scenario occurs when you have all 7 German books and 11 each of French, Spanish, Russian, and Italian. You still wouldn't have 12 of any language. That would be a total of 7+11+11+11+11=51 books. Adding one more book (it will either be Spanish, Russian, or Italian) will guarantee 12 of one language. Therefore, 51+1=52 books guarantees at least 12 of one language.

5. Suppose you have an 8"x8" piece of paper. Fold it in half both vertically and horizontally to make a 4"x4" square. You should have 9 rectangles when you unfold the paper. Now fold the piece of paper in half once more vertically and horizontally to make a 2"x2" square.
    a. How many rectangles would you have if you unfolded the paper?

See part b to see how I get 100.

    b. Explain how you would you tell your students to systematically count the rectangles.

Have the students count each of these type of rectangles.
1 x 1: 16
1 x 2: 24 (3 in each of 4 verticals and 3 in each of 4 horizontals)
1 x 3: 16 (2 in each of 4 verticals and 2 in each of 4 horizontals)
1 x 4: 8 (1 in each of 4 verticals and 1 in each of 4 horizontals)

2 x 2: 9
2 x 3: 12 (2 in each of 3 verticals and 2 in each of 3 horizontals)
2 x 4: 6 (1 in each of 3 verticals and 1 in each of 3 horizontals)

3 x 3: 4
3 x 4: 4 (1 in each of 2 verticals and 1 in each of 2 horizontals)

4 x 4: 1

    c. Fold it one more one time in half one more time, both horizontally and vertically (to make a 1"x1" square). Explain how you systematically count the rectangles this time.

Recall, when you folded the 2" x 8" piece of paper n times, there were $\frac{2^n(2^n+1)}{2}$ **rectangles. This means if you fold it one time, there were 3 rectangles; twice, you got 10 rectangles. The answers for when you fold the square piece of paper both horizontally and vertically end up being** $\left(\frac{2^n(2^n+1)}{2}\right)^2$**, where n is the number of folds in each direction. Thus, the answer will be** 36^2 **for 3 folds (n=3 here).**

    d. Without actually folding the paper, how many rectangles would there be if you folded again to make a ½" x ½" square?

Using the formula above, it will be 136^2 **for four folds in each direction (n=4 here).**