Lab 4

Simple Multiplication & Division Models for Whole Numbers

Objectives:

1. The teacher will construct appropriate models for basic whole number multiplication facts.
2. The teacher will distinguish between the two division settings--the number of equivalent subsets setting and the number of elements in a subset setting.
3. The teacher will construct appropriate models for basic whole number division facts.

Terms to Know

Factors → $4 \times 3 = 12$ ← Product

3 ← Quotient

Divisor → $4 \overline{)12}$ ← Dividend

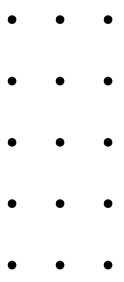
TOPIC: SIMPLE MODELS FOR WHOLE NUMBER MULTIPLICATION:

- ***Materials: PAPER AND PENCIL***

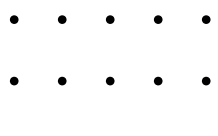
1. Arrays

Suppose we wish to demonstrate 3 x 5 with an array of dots. To do this we can use either a 5 by 3 or a 3 by 5 array of dots.

A 5 by 3 array for 5 x 3



A 3 by 5 array for 3 x 5

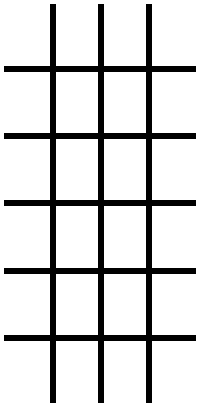


Sketch an array which will demonstrate 6 x 8.

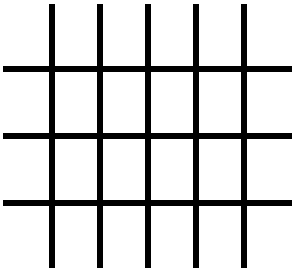
2. Crossing Lines

Some texts for elementary school mathematics use this model.

5 x 3 demonstrated by 5 by 3 crossing lines The product is the intersections of the lines



3 x 5 demonstrated by 3 by 5 crossing lines The product is the intersections of the lines



TOPIC: REPEATED ADDITION MULTIPLICATION MODELS

- ***Materials: CUISENAIRE RODS***

Many materials for children use repeated addition as a basis for determining products.
For example: 3 x 5 = 5 + 5 + 5 or 5 x 3 = 3 + 3 + 3 + 3 + 3

3. **Cuisenaire Rods--the white rod is one.**

Consider 3 x 5 again:

We first choose the LG and the Y rods. These are placed in a tower, vertical cross-position. That is:

Y =	LG = 3	5

Enough yellow rods are now placed adjacent to the yellow rod under the light green rod until you "fill" the light green rod with yellow rods.

Y	LG	
Y		
Y		

The light green rod is removed and you are left with 3 yellow rods

Yellow = 5
Yellow = 5
Yellow = 5

The 3 yellow rods are placed in a train. At this point we may still need to check basic addition. The orange and yellow rod train is the special train that shows place value.

Orange = 10		Yellow = 5
Yellow = 5	Yellow - 5	Yellow =5

3 x 5 = 10 + 5 = 15, The white rod is one
This also shows 3 x 5 as 5 + 5 + 5

NOTE: The order in which you do this does not matter. This model shows 3 x 5 and 5 x 3. Additionally, having children explore all of the various ways to show 3 x 5 and 5 x 3 is a great way for the kids to see that 3 x 5 = 5 x 3

a. As a group, use Cuisenaire Rods to model 6 x 5. Sketch a picture of your work here. Label everything. Use a Comparison Place Value Train to compute the final solution. What's one?

b. As a group, use Cuisenaire Rods to model 9 x 6. Sketch a picture of your work here. Label everything. Use a Comparison Place Value Train to compute the final solution. What's one?

c. As a group, use Cuisenaire Rods to model 4 x 11. Sketch a picture of your work here. Label everything. Use a Comparison Place Value Train to compute the final solution. What's one?

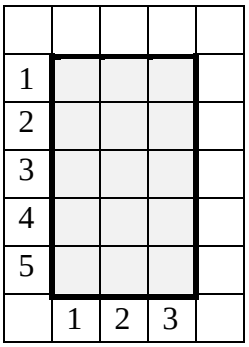
TOPIC: REPEATED ADDITION MULTIPLICATION MODELS

- ***Materials: PAPER AND PENCIL***

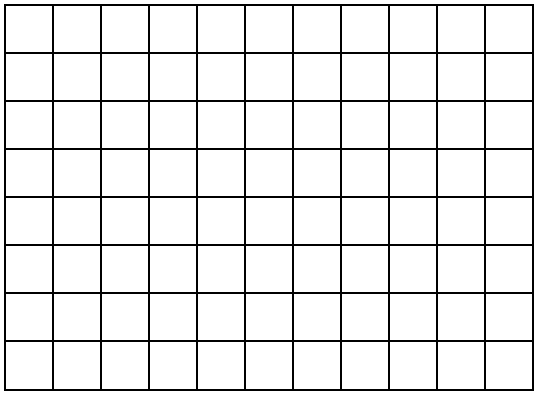
4. **Graph or squared paper.**

Another repeated addition model is squared paper. To demonstrate multiplication we mark one factor horizontally and the other factor vertically.

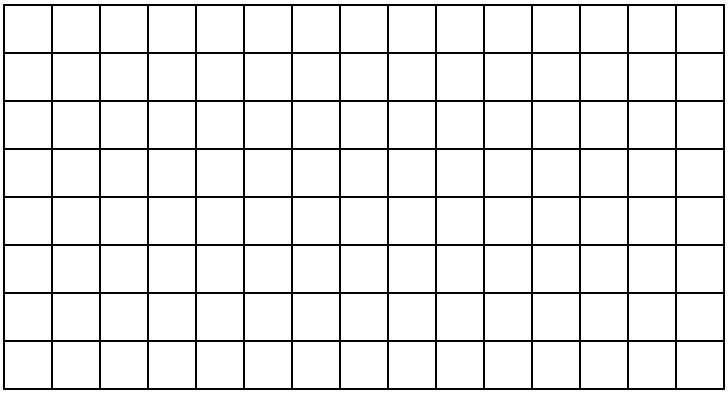
The shaded region shows 3 x 5 or 5 x 3. The order doesn't matter. Notice that we are using area here to show a product.



Model 4 x 7.



Model 12 x 5.



TOPIC: TWO DIVISION SETTINGS FOR WHOLE NUMBER DIVISION

- ***Materials: WOODEN CUBES***

There are two reasons to divide[1]:

Division Setting One--The Number of Equivalent Subsets:
This can be described as:

"If you know the original amount and the size of one share;
you divide to find the number of shares."

Given The size of the share = the number of elements in each subset.
&
Looking For The number of shares = the number of equivalent subsets

Division Setting Two--The Number of Elements in A Subset:
This can be described as:

"When you know the original amount and the number of shares;
you divide to find the size of each share. "

Given The number of shares = the number of subsets
&
Looking For The size of the share = the number of elements in each subset.

Materials: WOODEN BLOCKS

Although the next problems do not specifically refer to using blocks, use modeling with the blocks. I.e.: Blocks as cookies and blocks as apples. Each person in your group should have their own model.

1. Which setting matches
   "12 cookies are divided evenly among 3 boys. How many cookies does each boy get?"

   The original amount is: 12 cookies The size of a share is: _____ The number of shares is: _____

   So the setting is: # of Equivalent Subsets (= solution) or # of Elements (= solution)
   Circle one of the settings

2. Which setting matches
   "Some boys are given 3 cookies each. If there are 12 cookies, how many boys get cookies?"

   The original amount is: 12 cookies The size of a share is: _____ The number of shares is: _____

   So the setting is: # of Equivalent Subsets (= solution) or # of Elements (= solution)
   Circle one of the settings

[1] "Math at Hand: A Mathematics Handbook," Great Resource Publishing Company

3. Use the pictures (below) to fill out the following (everyone in your group should have their own block model.):

- I have 12 apples and I give 4 apples each to some friends. How many friends get apples?

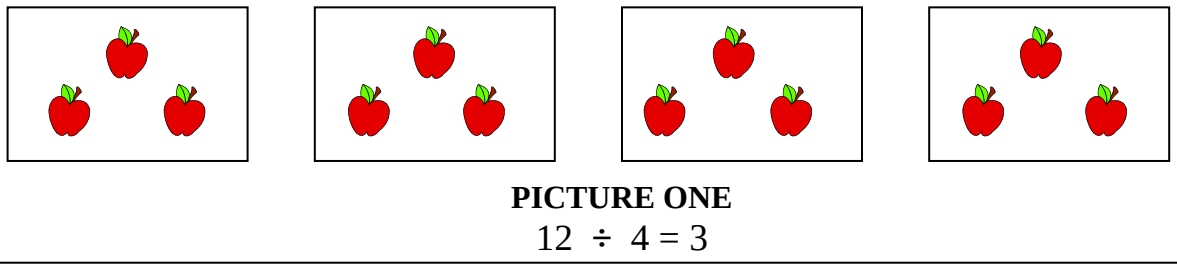
This is Picture ONE / TWO and & Division Setting # SUBSETS / # ELEMENTS
Circle one ***Circle one***

We are looking for the number of friends as the answer to our division sentence.

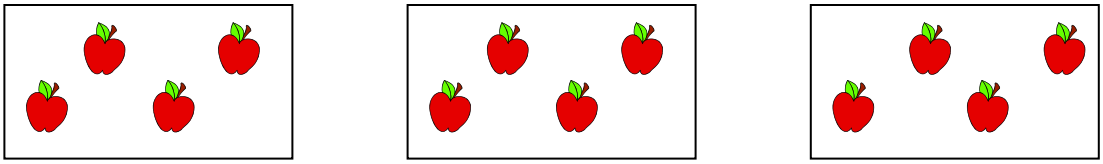
- I have 12 apples and 4 friends divide them evenly. How many apples does each friend get?

This is Picture ONE / TWO and & Division Setting # SUBSETS / # ELEMENTS
Circle one ***Circle one***

We are looking for the number of apples as the answer to our division sentence.



PICTURE ONE
12 ÷ 4 = 3



PICTURE TWO
12 ÷ 4 = 3

4.

a. Write a problem to ask 20 ÷ 10 = _____ in the Number of Equivalent Subsets division setting and circle the ***X***s to show your problem.

X	***X***	***X***	***X***	***X***	***X***	***X***	***X***	***X***	***X***
X	***X***	***X***	***X***	***X***	***X***	***X***	***X***	***X***	***X***

b. Write a problem to ask 20 ÷ 10 = _____ in the Number of Elements division setting and circle the ***X***s to show your problem.

X	***X***	***X***	***X***	***X***	***X***	***X***	***X***	***X***	***X***
X	***X***	***X***	***X***	***X***	***X***	***X***	***X***	***X***	***X***

TOPIC: SIMPLE MODELS FOR WHOLE NUMBER DIVISION

- ***Materials: WOODEN BLOCKS***

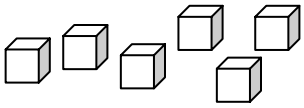
5. Modeling $6 \div 3 =$ _____? with blocks

- ***Define One***

A reasonable model for 1 is one block..

- ***Define Six***

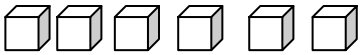
Since 1 is a one block, 6 is six blocks.



Now there are TWO situations that both result in the division sentence $6 \div 3 =$ _____?

Situation One:

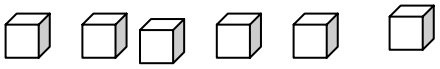
How many groups of three blocks are there in a group of six blocks?



Which division setting is this? _____ Circle the blocks to show this.

Situation Two:

If I place my six blocks into three equally sized piles, how many blocks are in each pile?



Which division setting is this? _____ Circle the blocks to show this.

Notice that this is no different than the "apple" questions above. We're just using blocks instead of apples. We could use any small counter here.

- ***Materials: CUISENAIRE RODS***

- Modeling $20 \div 4 =$ _____? with Cuisenaire Rods
- ***Define One***

A reasonable model for 1 is the white rod.

White Rod is 1

- ***Define Twenty***

Since 1 is the white rod, 20 is an orange-orange train.

This train is 20

Orange = 10	Orange = 10

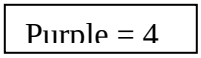
Now there are TWO situations that both result in the division sentence $20 \div 4 =$ _____?

SITUATION ONE:

FIRST: COLORED RODS

- ***Define Four as a colored rod***

Since 1 is the white rod, 4 is a purple rod.



We can ask a question ***in terms of colored rods:***

"How many times does the purple rod go into the orange-orange train?"

ORANGE			ORANGE	
PURPLE	PURPLE	PURPLE	PURPLE	PURPLE

Which division setting is this? _____

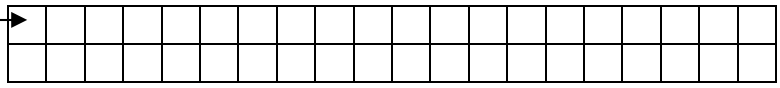
STILL SITUATION ONE:
SECOND: WHITE RODS ONLY

We can ask a question ***in terms of white rods only (which is the same idea as plain cubes):***

"How many groups of 4 white rods are there in a group of 20 white rods?

- Mark the second row of the blocks to show to show $20 \div 4$ in this setting, use only white rods

This is 20 white rods →



Which division setting is this? _____

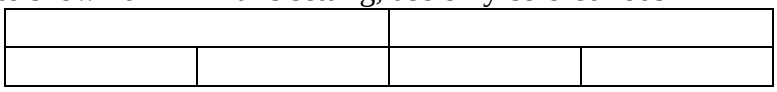
SITUATION TWO:

The OTHER division setting is: _____

FIRST: COLORED RODS

Since 1 is the white rod, 20 is an orange-orange train.

- Label this picture to show $20 \div 4$ in this setting, use only colored rods



As a group, determine how to ask a question ***in terms of colored rods*** that would result in the previous question.

We want our ANSWER to be the length (color) of a rod. Remember, you can't mention 5 or the color of the rod that matches 5 in your question.

COLORED ROD QUESTION:

Which division setting is this? _____

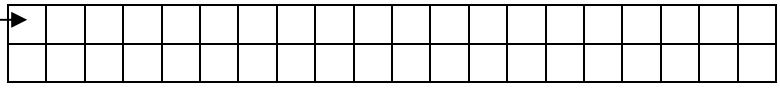
STILL SITUATION TWO:
SECOND: WHITE RODS ONLY

As a group, determine how to rephrase your question ***in terms of white rods only: Don't mention the colored rods or any color other than white.***

WHITE ROD QUESTION:

- Mark the second row of the blocks to show to show $20 \div 4$ in this setting, use only white rods

This is 20 white rods →



Which division setting is this? _____

6. Determine the following as a group

- Write four questions, two questions for each division setting as follows.
- For each question, model the problem, sketch and label here.

FIRST:

- Create the two possible colored Cuisenaire Rod Models for $36 \div 12 =$ _____ for reference.

Division Setting, Number of Elements $36 \div 12 =$ _____

i. COLORED CUISENAIRE RODS

Question:

Sketch*:*

ii. WHITE (ONLY) CUISENAIRE RODS

Question:

Sketch*:*

Division Setting, Number of Subsets $36 \div 12 =$ _____

i. COLORED CUISENAIRE RODS

Question:

Sketch*:*

ii. WHITE (ONLY) CUISENAIRE RODS

Question:

Sketch*:*

7. **Graph or squared paper.**

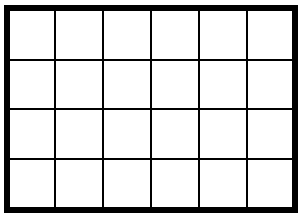
Your turn! Write two problems that would result in the division sentence:

$24 \div 4 =$ _____?

Write one problem for each division setting that uses drawing on graph paper. Be sure to use words and phrases like "4 squares," "4-square region" and "24 square region" in your problems:

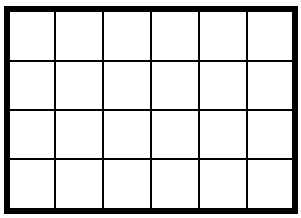
Your Question / Division Setting, # Subsets

Your Picture:



Your Question / Division Setting, # Elements

Your Picture:



LAB FOUR DISCUSSION QUESTIONS

As a group, discuss and answer the following questions. Feel free to also discuss with the other groups.

1. Name other manipulative that could be used for SIMPLE MULTIPLICATION & DIVISION MODELS FOR WHOLE NUMBERS.

2. What mathematical knowledge would you want your students to KNOW prior to introducing SIMPLE MULTIPLICATION & DIVISION MODELS FOR WHOLE NUMBERS?

Note: In the elementary school classroom, the primary study is how to divide, not identifying Division settings. As you saw in the Great Source books in the classroom, the various methods of running across a division problem are pointed out to the children. However, this is just to alert them to the idea that division can arise in more than one way. It does not make sense teach children how to identify division settings--just their teachers!

3. What mathematical knowledge would you want to EMPHASIZE while introducing SIMPLE MULTIPLICATION & DIVISION MODELS FOR WHOLE NUMBERS?

4. Suppose you ask a student to divide 154 by 11. How can the student check his/her answer (without using a calculator or checking with their neighbors)? (What is the "inverse" operation of division?)

5. Which manipulative do you feel is the most effective for introducing Multiplication and Division? Could you see yourself using any of these manipulatives to introduce Multiplication and Division in YOUR classroom?