Pre-Calculus Review Worksheet Answers

1. Write the equation of the line with slope 5 that passes through $(-3, 7)$.

$$y = 5(x + 3) + 7$$

2. Write the equation of the line with slope -4 and has a y-intercept of 9.

$$y = -4x + 9$$

3. Write the equation of the line that passes through the points $(-2, 7/11)$ and $(18, 14)$.

$$y = \frac{147}{220}(x + 2) + \frac{7}{11}$$

4. Write the equation of the line that is perpendicular to the line $y - 3x = 5$ and has y-intercept -5.

$$y = -\frac{1}{3}x - 5$$

5. Write the equation of the line that is parallel to the line $3y - 6x + 7 = 0$ and passes through the point $(2, 1)$.

$$y = 2(x - 2) + 1$$

6. Find a point (a, b) so that the line through (a, b) and $(-2, 7)$ is perpendicular to the line through $(-2, 7)$ and $(4, 9)$.

   Find any point on the line $y = -3(x + 2) + 7$.

7. State the domain of $g(x) = \sqrt{x}/(x^2 + x - 6)$.

   We cannot have the denominator being zero and we need $x \geq 0$. Therefore our answer is $x \in [0, 2) \cup (2, \infty)$.

8. State the domain of $f(x) = \sqrt{x^2 - 4x + 5}$.

   Since $x^2 - 4x + 5 \geq 0$ for all x, the domain is all real numbers.

9. Sketch the graph of the function defined by $x^3 - 1$.

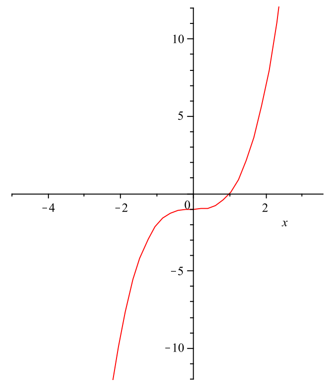

Figure 1: $x^3 - 1$

10. Plot several points and sketch the graph of the function defined by the expression:

$$h(x) = \begin{cases} x^2 & \text{if } x < 1 \\ 2 - x^2 & \text{if } x \geq 1 \end{cases}$$

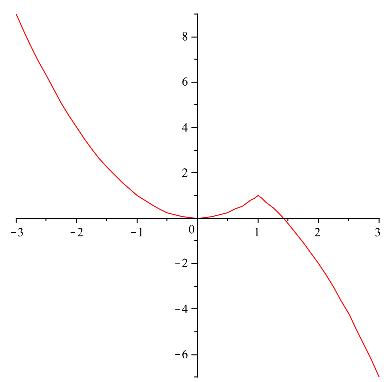

Figure 2: Piecewise Function

11. Let $F(x) = x^2 + 5$ and $G(x) = (x+1)/(x-1)$. Find (a) $F - 3G$ and (b) $F \circ G$.

$$\text{(a) } F - 3G = x^2 + 5 - \frac{3(x+1)}{x-1} \qquad \text{(b) } F \circ G = \frac{(x+1)^2}{(x-1)^2} + 5$$

12. Write the function $h(x) = \sqrt{x-7}$ as the composition of two functions.

$$f(x) = x - 7 \quad g(x) = \sqrt{x} \quad h(x) = g \circ f(x)$$

13. Write the following polynomials as a product of irreducible polynomials of degree one or two.

$$\text{(a) } x^3 + x^2 - 4x - 4 \quad \text{(b) } x^4 + 3x^2 + 2$$

$$\text{(a) } (x+1)(x-2)(x+2) \qquad \text{(b) } (x^2+2)(x^2+1)$$

14. Calculate the following expressions without using a calculator:

$$\text{(a) } \cos(2\pi/3)\csc(2\pi/3) \quad \text{(b) } 4\tan(\pi/4) - \sin(17\pi/2)$$

$$\text{(a) } \frac{-1}{2} \cdot \frac{2}{\sqrt{3}} = -\frac{1}{\sqrt{3}} \qquad \text{(b) } 4 \cdot 1 - 1 = 3$$

15. Suppose θ is a number between 0 and $\pi/2$ and $\sin(\theta) = 1/3$, determine $\cos(\theta)$.

$$\cos^2(\theta) + \sin^2(\theta) = 1 \Longrightarrow \cos^2(\theta) = \frac{8}{9} \Longrightarrow \cos(\theta) = \frac{2\sqrt{2}}{3} \text{ in QI.}$$

16. Suppose θ is a number between 0 and $\pi/2$ and $\cos(\theta) = 3/5$, determine $\tan(\theta)$.

$$\cos^2(\theta) + \sin^2(\theta) = 1 \Longrightarrow \sin^2(\theta) = \frac{16}{25} \Longrightarrow \sin(\theta) = \frac{4}{5} \text{ in QI.}$$

Thus,

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{4}{3}.$$

17. State which of the six trigonometric functions are positive when evaluated at θ where $\theta \in (3\pi/2, 2\pi)$.

$$\cos(\theta), \sec(\theta)$$

18. Use one or more of the basic trigonometric identities to derive the given identities:

(a)

$$\tan(\theta + \phi) = \frac{\tan(\theta) + \tan(\phi)}{1 - \tan(\theta)\tan(\phi)}$$

$$\text{LHS} = \frac{\sin(\theta+\phi)}{\cos(\theta+\phi)} = \frac{\sin(\theta)\cos(\phi) + \sin(\phi)\cos(\theta)}{\cos(\theta)\cos(\phi) - \sin(\theta)\sin(\phi)}$$

$$\text{RHS} = \frac{\frac{\sin(\theta)}{\cos(\theta)} + \frac{\sin(\phi)}{\cos(\phi)}}{1 - \frac{\sin(\theta)}{\cos(\theta)}\frac{\sin(\phi)}{\cos(\phi)}} = \frac{\frac{\sin(\theta)\cos(\phi)+\sin(\phi)\cos(\theta)}{\cos(\theta)\cos(\phi)}}{\frac{\cos(\theta)\cos(\phi)-\sin(\theta)\sin(\phi)}{\cos(\theta)\cos(\phi)}} = \frac{\sin(\theta)\cos(\phi) + \sin(\phi)\cos(\theta)}{\cos(\theta)\cos(\phi) - \sin(\theta)\sin(\phi)} = \text{LHS}$$

(b)

$$\sin(\theta) = \cos(\pi/2 - \theta)$$

$$\text{RHS} = \cos(\pi/2)\cos(\theta) + \sin(\pi/2)\sin(\theta) = 0 + 1 \cdot \sin(\theta) = \sin(\theta)$$

(c)

$$\cos(\theta + \pi) = -\cos(\theta)$$

$$\text{LHS} = \cos(\theta)\cos(\pi) - \sin(\theta)\sin(\pi) = -\cos(\theta) - 0 = -\cos(\theta)$$