Chapter 10: Exponential functions

10.1 INTEGER EXPONENTS
10.2 FRACTIONAL EXPONENTS

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Integer

- “The numbers”
- Number that has no fractions. Can be written without decimal or rational component.
- ...— 3, — 2, — 1, 0, 1, 2, 3...
- “...” means “continuing on in the same pattern without end”

Exponent

- The number of times the base is taken as a factor
- Don’t apply exponent to values not part of base
- Base includes numbers or variables attached to the exponent directly
- There may be factors of the exponential that are not part of the base
- Watch for ( ) or not

Negative exponent

- May or may not be a negative number
- Solve by using quotient rule of exponents

$$\frac{b^3}{b^5} = b^{3-5} = b^{-2}$$

Negative exponent

- Solve by using quotient rule of exponents

$$\frac{b^3}{b^5} = b^{3-5} = b^{-2}$$

- Implies moving base across fraction bar because the two are equivalent

$$\frac{b^3}{b^5} = \frac{bbb}{bbbbb} = \frac{1}{bb} = \frac{1}{b^2}$$

Can only combine LIKE bases

$$\frac{b^{-4}}{b^{3}} = b^{-4-3} = b^{-7} = \frac{1}{b^{7}}$$

$$\frac{b^{-4}}{c^{3}} = \frac{1}{b^{4}c^{3}}$$

Simplify Expressions with Exponents

- No parentheses
- Any values are combined into single value without exponents
- Any variable appears only a single time
- Every exponent is positive

No parentheses

- Rule:
  - Multiply the exponent inside by the one outside

$$\left(b^{-3}\right)^4 = b^{-12} = \frac{1}{b^{12}}$$

- Outside exponent means the base, inside, is the factor to be taken that many times

$$\left(b^{-3}\right)^4 = \left(b^{-3}\right)\left(b^{-3}\right)\left(b^{-3}\right)\left(b^{-3}\right) = b^{-12} = \frac{1}{b^{12}}$$

No parentheses

- Deal with the outside negative first

$$\left(2b^{-5}\right)^{-3} = \frac{1}{\left(2b^{-5}\right)^{3}}$$

No parentheses

- Multiply the exponents inside by the one outside

$$\frac{1}{\left(2b^{-5}\right)^{3}}=\frac{1}{2^{3}b^{(-5)3}}$$

Values reported without exponents

$$\frac{1}{2^3 b^{-15}} = \frac{1}{8b^{-15}}$$

$$\frac{1}{8b^{-15}} = \frac{b^{15}}{8}$$

- And no negative exponents

Can be very tedious when complicated

- Steps are not hard
- Keep track of where you are

$$\left(\frac{18b^{-4}c^{7}}{6b^{-3}c^{2}}\right)^{-4}$$

- Reduce inside first

$$=\left(3b^{-4-(-3)}c^{7-2}\right)^{-4}=\left(3b^{-1}c^{5}\right)^{-4}$$

Now deal with ( )

- Multiply each inside exponent by the outside exponent

$$\left(3b^{-1}c^{5}\right)^{-4} = 3^{-4}b^{(-1)(-4)}c^{5(-4)}$$

Then deal with negative exponents

- Any base with a negative exponent needs to be moved across the fraction bar

$$3^{-4}b^{4}c^{-20}=\frac{b^{4}}{3^{4}c^{20}}=\frac{b^{4}}{81c^{20}}$$

- And the numerical exponent needs to calculated

Scientific notation

- Move decimal point to have a single digit to left of it
- Multiply by power of 10 to make it equivalent expression
- Common error is to raise number to power instead of 10 to power

7×10^{3} 7×10^{-3}

- 7 x 1000 = 7,000
- Move decimal point three places to right

- 7 x 1/1000 = 0.007
- Move decimal point three places to left

- Think about a number line!!

845,000,000 0.0000382

- 8.45×10^{8}
- 8 from how many decimal places
- 10^{8} is a very large number...
- 3.82×10^{-5}
- 5 from how many decimal places
- 10^{-5} is a very small number...

778,000,000   0.000012

- 7.78×10^8
- 8 from how many decimal places
- Did you apply the correct sign of exponent
- 10^8 is a very large number...LOOK!!
- 1.2×10^{-5}
- 5 from how many decimal places
- 10^{-5} is a very small number...LOOK!!

Rational exponents: fractions in exponent!!

- Follow all the same rules for exponents
- Do not assume fractional exponent is a fractional number!!
- Denominator of exponent means a root

Rules for exponents: page 594

- The last rule: doesn't matter if n gets switched with m
- Important note for the rational exponents

$$b^m b^n = b^{m+n}$$

$$\frac{b^m}{b^n} = b^{m-n}$$

$$(bc)^n = b^n c^n$$

$$\left(\frac{b}{c}\right)^n = \frac{b^n}{c^n}$$

$$(b^m)^n = b^{m \cdot n}$$

Rational exponent $9^{(1/2)}$

- Note you can write 9 as 3^2
- Apply rule for nested exponents

$$9^{\left(\frac{1}{2}\right)} = \left(3^2\right)^{\left(\frac{1}{2}\right)} = 3^{2\left(\frac{1}{2}\right)} = 3^1 = 3$$

Watch out for negative base

- If root is odd, retain the negative

$$(-64)^{1/3} = \left[(-4)^3\right]^{1/3} = -4$$

- If root is even, it is not a real number

$$(-16)^{1/4} = \left[(-2)^4\right]^{1/4} = 2$$

- But $2^4 = 16$ so above is not true