8.5 Factoring to Solve Polynomial Equations

Zero Factor Property (ZFP)

- Multiply by zero: result equals zero
- Factor a quadratic expression equal to zero
  - You have two factors whose product is zero
  - One of the factors must be zero
- Find values for variable that make this true
- These are the x-intercepts of the graph of the equation

$(x - 5)(x + 2)=0$

- Equals zero
- Use Zero factor property (ZFP)
- $(x - 5)=0$
- $+5 \quad +5$
- $x=5$
- $(x + 2)=0$
- $-2 \quad -2$
- $x = -2$
- Check! In ORIGINAL

$x^2 - 2x - 8 = 0$

- Equals zero
- If it factors, use zero factor property (ZFP)
- $(x+2)(x - 4)=0$
- Set each factor equal to zero, solve each
- $(x + 2)=0 \qquad x - 4 = 0$
- $-2 \quad -2 \qquad +4 \quad +4$
- $x = -2 \qquad x = 4$
- Check each in ORIGINAL!

$x^2+8x+15=0 \rightarrow (x+3)(x+5)=0$

- $(x+3)=0$
- Solve for x by subtracting 3 from both sides: *WRITE THIS STEP*
  - $x + 3 = 0$
  - $-3 \quad -3$
  - $x= -3$
- And $(x+5)=0$
- Solve for x: Subtract 5 from both sides: *WRITE THIS STEP*
  - $x + 5 = 0$
  - $-5 \quad -5$
  - $x= -5$

CHECK EACH VALUE in ORIGINAL EQUATION: $x^2+8x+15=0$

- $x=-3$
- $(-3)^2+8(-3)+15=$
- $9-24+15=0$
- Or
- $x=-5$
- $(-5)^2+8(-5)+15=$
- $25-40+15=0$
- Write: $x=-3,-5$ this is not an ordered pair

$4x^2 + 26x + 30 = 0$

- Equals zero
- If it factors, use zero factor property (ZFP)
- Has Greatest Common Factor: GCF = 2
- $2(2x^2 + 13x + 15) = 0$
- Now factor trinomial in ()
  - $2 \cdot 15 = 30$
  - $30 = 10 \cdot 3$
  - $10 + 3 = 13$
- $2[(2x^2 + 10x) + (3x + 15)] = 0$
- $2[2x(x + 5) + 3(x + 5)] = 0$
- $2(x + 5)(2x + 3) = 0$
- Set each factor equal to zero, solve each
- $(x + 5) = 0$ $\quad 2x + 3 = 0$
- $\quad -5 \ -5 \quad\quad -3 \ -3$
- $x = -5$ $\quad 2x = -3 \rightarrow x = -3/2$
- Check each in ORIGINAL!

$x^2+2x=0$

- $x(x+2) = 0$ factored: set each factor = 0
- $x=0$ this IS a solution…be sure to include it
- $x+2=0$ solve for x
- by subtracting 2 from both sides
- $x=-2$
- Check: $(0)^2+2(0)=$ $\quad (-2)^2+2(-2)=$
- $\quad 0+0=0 \quad\quad 4-4=0$

$6x^2 = 18x$

- Needs to be = 0 to use ZFP
- $6x^2 - 18 = 0$
- Greatest common factor: 6x
- $6x(x - 3) = 0$
- Set each factor with variable = 0
  - $x = 0$ $\quad x - 3 = 0$
  - $\quad + 3 \quad + 3$
  - $\quad x = 3$
- Check both in original:
  - $6(0)^2 = 0$, $18(0) = 0$, so 0 is good
  - $6(3)^2 = 54$, $18(3) = 54$, so 3 is good

$2x^2 - 8x = 5x - 20$

- Not equal to zero, so ZFP cannot be used
- Until you MAKE it equal to zero!!
- Move terms by adding or subtracting on both sides
- $2x^2-8x-5x+20=0$
- combine like terms: $2x^2-13x+20=0$
- Factor: $(2x-5)(x-4)=0$
- Use ZFP: $2x-5=0$ $\quad\quad x-4=0$
- $\quad 2x=5$, $x=5/2 \quad\quad x=4$

$2x^2 - 8x = 5x - 20$

- Check $x=5/2$
- Check $x=4$

$$2[4]^2 - 8[4] ? = 5[4] - 20$$
$$2[16] - 32 ? = 20 - 20$$
$$32 - 32 = 0$$
$$20 - 20 = 0$$

$$2\left[\frac{5}{2}\right]^2 - 8\left[\frac{5}{2}\right] ? = 5\left[\frac{5}{2}\right] - 20$$
$$2\left[\frac{25}{4}\right] - \frac{40}{2} ? = \frac{25}{2} - \frac{40}{2}$$
$$\frac{25}{2} - \frac{40}{2} = -\frac{15}{2}$$

$(x + 2)(x - 4) = 7$

- Need to make it equal zero to use ZFP
- BE CAREFUL!!
- If you subtract 7 from both sides, it does
- (x+2)(x-4)-7=0,
- but it is not composed of factors,
- so you cannot use zero factor property
- What could be done?

$(x + 2)(x - 4) = 7$

- Use FOIL on left first:
- $x^2-2x-8=7$
- Now subtract 7 from both sides
- $x^2-2x-15=0$, and factor
- $(x-5)(x+3)=0$
- $x-5=0$ $x+3=0$
- Add 5 to both subtract 3 from both
- $x=5$ $x=-3$ and check

$f(x) = x^2 - 3x - 23$

- find f(5)
- Input!!
- f(5) Means: put 5 in for x, find value of function
- $f(5) = (5)^2 - 3(5) - 23$
- $f(5) = 25 - 15 - 23 = -13$

$f(x)=x^2-3x-23, f(x)=5$

- f(x) = 5
- Output!!
- f(x) = 5 Means $5 = x^2 - 3x - 23$
- Use Zero Factor Property to solve
- Subtract 5 from both sides
- $0 = x^2 - 3x - 28$
- Factor: $(x - 7)(x + 4) = 0$
- $x - 7 = 0$ $x + 4 = 0$
- $x = 7$ $x = -4$