Test Content — Mathematics (4IMF)

The Major Field Test in Mathematics consists of 50 questions, some of which may be grouped in sets and based on such materials as diagrams and graphs. The questions are drawn from the courses of study most commonly offered as part of an undergraduate mathematics curriculum. Programs can choose when and where to administer the tests. It is designed to take two hours and may be split into two sessions. This test must be given by a proctor. Mathematical operations do not require the use of a calculator.

The outline below shows the content areas covered on the test and the approximate distribution of questions among the areas.

The Test Outline

I. Calculus (~30%): includes the usual material from three semesters of:

A. Calculus

B. Single-variable calculus

C. Multivariable calculus

D. Separable differential equations

II. Algebra (~30%)

A. Linear Algebra

1. Matrices
2. Linear transformations
3. Eigenvalues and eigenvectors
4. Vector spaces
5. Systems of linear equations

B. Abstract Algebra

1. Elementary theory of groups, rings and fields
2. Elementary topics from number theory

III. Additional Topics (~40%)

A. Complex analysis

B. Differential equations

C. Discrete mathematics (including graph theory and combinatorics)

D. Foundations (including logic, proofs, sets, functions and relations)

E. Geometry

F. Point-set topology

G. Probability and statistics

H. Real analysis

The relative percentages of mathematics questions at various cognitive levels are listed below:

I. Routine (~55%): Includes only two or three definitions and no more than a two-step reasoning process, or involves standard techniques normally taught and practiced extensively in a course that is generally required or strongly recommended for all math majors at most institutions.

II. Nonroutine (~25%): Includes all items that are considered insightful. Also includes items that require several steps of reasoning and items that require either the use of several definitions or a new definition that the student would not be expected to know. Some questions may require bringing techniques from two or more areas to bear on one problem (e.g., treating differentiable functions as elements of an algebraic system).

III. Applied (~20%): There is conceptual overlap between the cognitive-level categories of applied, routine, and non-routine. The general nature of the question will determine the category for the question. For example, all questions with real-world settings are placed in the applied category. On the other hand, questions involving standard applications of one area of mathematics to another, such as using differential calculus to solve geometric problems, would not be placed in the applied category.

How scores for the Major Field Test in Mathematics are reported

Total Score – Reported for each student and summarized for the group.

Assessment Indicators – Reported for the group* only.

– Calculus (15)
– Algebra (15)
– Routine (27–28)
– Nonroutine (12–13)
– Applied (10)

Numbers in parentheses are the approximate number of questions in each category.

* A minimum of five (5) students is required for assessment indicators to be reported.

MATHEMATICS TEST
SAMPLE QUESTIONS

The following questions illustrate the range of the test in terms of the abilities measured, the disciplines covered, and the difficulty of the questions posed. They should not, however, be considered representative of the entire scope of the test in either content or difficulty. An answer key follows the questions.

1. On a questionnaire, a respondent must choose 3 of the 5 questions presented. How many different combinations of 3 questions can the respondent possibly choose?

   (A) 10
   (B) 15
   (C) 20
   (D) 30
   (E) 60

2. A function f has the property that, at every point (x, y) on the curve $y = f(x)$, the slope of the line tangent to the curve is equal to $2xy$. Which of the following best describes the function f ?

   (A) Linear
   (B) Trigonometric
   (C) Inverse trigonometric
   (D) Logarithmic
   (E) Exponential

3. Let A and B be metric spaces and let $f : A \to B$. Suppose that whenever X is an open set in B, the set $\{a \in A : f(a) \notin X\}$ is closed in A. Which of the following must be true?

   I. f is injective
   II. f is continuous.
   III. f is a homeomorphism.

   (A) None
   (B) II only
   (C) III only
   (D) I and III only
   (E) I, II, and III

4. In the xy-plane, the line that is tangent to the graph of $y = x^2$ at the point $(2, 4)$ has a slope of

   (A) $\frac{1}{2}$
   (B) 1
   (C) 2
   (D) 4
   (E) 8

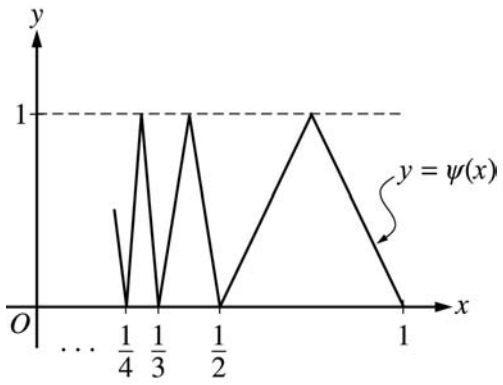
5. The set $\{1, 2, 4, 7, 8, 11, 13, 14\}$ forms a group under the operation of multiplication modulo 15. Which of the following is the cyclic subgroup generated by $\{7\}$?

   (A) $\{1, 7\}$
   (B) $\{1, 2, 7, 14\}$
   (C) $\{1, 4, 7, 13\}$
   (D) $\{1, 7, 8, 13, 14\}$
   (E) $\{4, 7, 11, 14\}$

6. For each real number $t \neq 0$, define the function $\phi_t : \mathbb{R} \to \mathbb{R}$ by $\phi_t(x) = |x|^{|t|}$. A subset A of real numbers is called invariant with respect to the collection of functions ϕ_t if $\phi_t(A) \subseteq A$ for each $t \neq 0$. For this collection of functions, which of the following intervals are invariant?

   I. $(0, 1]$
   II. $\left[0, \frac{1}{2}\right)$
   III. $(0, \infty)$

   (A) I only
   (B) II only
   (C) III only
   (D) I and III
   (E) II and III



7. A portion of the graph of a continuous nonnegative function $y = \psi(x)$ is shown above, where $\psi(0) = 0$ and $\psi\left(\frac{1}{n}\right) = 0$ for each positive integer n. If the graph of $y = \psi(x)$ between $x = \frac{1}{n+1}$ and $x = \frac{1}{n}$ consists of the congruent sides of an isosceles triangle of height 1 for each positive integer n, then $\int_0^1 \psi(x)\,dx =$

(A) $\frac{1}{2}$

(B) $\frac{1}{4}$

(C) $\frac{1}{\pi}$

(D) $\frac{1}{e}$

(E) $\frac{2}{e}$

8. The function f is differentiable on the interval $0 < x < 4$. If $f(1) = 1$ and $f(3) = 7$, then for some $1 < c < 3$, $f'(c)$ must be equal to

(A) 1
(B) 2
(C) 3
(D) 4
(E) 6

9. If a and b are integers, how many matrices of the form $\begin{bmatrix} 2 & a \\ b & 3 \end{bmatrix}$ are <u>not</u> invertible?

(A) One
(B) Two
(C) Four
(D) Eight
(E) More than eight

10. If V_n is the real vector space of all n-tuples of real numbers for each $n > 1$, which of the following must be true?

I. Every basis of V_n contains exactly n vectors.
II. Every basis of V_n is an orthogonal set of vectors.
III. Every set of $n + 1$ vectors of V_n is a linearly dependent set.

(A) I only
(B) II only
(C) I and II
(D) I and III
(E) II and III