

Problems of the Week (POWs) Assignments

Instructions: Your solution will be graded according to the official Oregon Scoring Guide, and thus your write up should include all your reasoning, and not just a final answer.

*The **emphasis** of these assignments is on **clear, complete and precise explanations**. Please write your solutions in a way that a "typical" student in the 5-8th grade range will be able to follow it.*

If you do not like the score you receive on your POWS, you could choose another one from each of the 5 assignments and turn in your second attempt by 3/7. Be sure to attach the POW you wish to replace. The scores of the 1st two attempts will then be averaged.

Some of these problems were taken from the following sources:

www.eduplace.com, <http://www.cmc.uwaterloo.ca/english/contests/pascal.shtml>

A. POW 1 Due 1/18: Choose one of the following two problems:

Three rugs have a combined area of 200m^2 . By overlapping the rugs to cover a floor area of 140m^2 , the area which is covered by exactly two layers of rug is 24m^2 . What area of floor is covered by three layers of rug?

P, Q, R, S, and T are five different integers between 2 and 19 inclusive.

- P is a two-digit prime number whose digits add up to a prime number.
- Q is a multiple of 5.
- R is an odd number, but not a prime number.
- S is the square of a prime number.
- T is a prime number that is also the mean (average) of P and Q.

Find P, Q, R, S, and T.

B. POW 2 Due 1/25: Choose one of the following two problems:

A bag contains eight yellow marbles, seven red marbles, and five black marbles. Without looking in the bag, Igor removes N marbles all at once. If he is to be sure, no matter which choice of N marbles he removes, there are at least four marbles of one color and at least three marbles of another color left in the bag, what is the maximum possible value of N?

Matt has a vinyl place mat that consists of a 10 by 10 grid. He thinks it has 100 squares, but then his older brother comes along and remarks, "Take another look. That mat has at least three times as many squares as you think!" How many squares does Matt's mat have?

C. POW 3 Due 2/1: Choose one of the following two problems:

Carmen, Jordy, Mariah, and Ronnie each have less than \$1.00 to spend at the snack bar. Altogether they have \$3.26, all in coins, including 7 quarters, 1 penny, and the same number of dimes as nickels.

- Each person has 7 coins.
- They each have a different number of dimes.
- Jordy has more dimes but fewer quarters than anyone else.
- Ronnie has the most money, and Jordy has the least.
- Mariah is the only one who has a penny.
- Ronnie has the same number of nickels as Jordy and Carmen has the same number as Mariah.
- Ronnie has the same number of nickels as quarters, but more dimes than nickels.

What combination of coins, and how much money, does each person have?

Find the sum of all positive integers less than 1,000 that are divisible by 3 but not by 2.

D. POW 4 Due 2/8: Choose one of the following two problems:

Shilpa gets “digits” from Raj at the local watering hole. She remembers that his 10-digit phone number is something like 1472583690, where it consists of one each of the digits 0-9. She also remembers that if the 1st digit (1 in this case) is a multiple of 1. The 1st two digits (14 in this case) is a multiple of 2. The 1st three digits (147 in this case) is a multiple of 3 and so on (...the 1st 10 digits are a multiple of 10). What is Raj’s phone number? There is a unique phone number which satisfies the above criteria.

How many leap years will there be from the years 2012 to 3004 inclusive?

- (i) Year Y is not a leap year if Y is not divisible by 4.
- (ii) Year Y is not a leap year if Y is divisible by 100 but not by 400.
- (iii) Otherwise year Y is a leap year.

E. POW 5 Due 2/15: Choose one of the following two problems:

How many four-digit positive integers x are there with the property that x and $3x$ have only even digits? (One such number is $x = 8002$, since $3x = 24006$ and each of x and $3x$ has only even digits.)

A palindrome is a positive integer whose digits are the same when read forwards or backwards. There are pairs of four-digit palindromes whose sum is a five-digit palindrome. One such pair is 2882 and 9339. How many such pairs are there?