MATH 396 Final Exam Review SOLUTIONS

1. Dale, Al and Mario each have a battery-operated car. The cars always travel in a straight line at a constant rate of speed. They decided to have a race. Each car started at the same time at the beginning of a straight race course.

   When Al's car crossed the finish line, it was ahead of Dale's car by 24 inches and was ahead of Mario's car by 32 inches. When Dale's car crossed the finish line, it was ahead of Mario's car by 10 inches. How many inches long is the race course?

<table>
<tr><td colspan="5">AJ</td></tr>
<tr><td colspan="2">Dale</td><td>8</td><td>8</td><td>8</td></tr>
<tr><td>Mario</td><td>8</td><td>8</td><td>8</td><td>8</td></tr>
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<table>
<tr><td colspan="2">Dale</td></tr>
<tr><td>Mario</td><td>10</td></tr>
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If Dale goes 24 inches and Mario goes 22 inches, Mario is 2 inches behind for every 24 inches Dale goes.

First round, Mario is 8 inches behind Dale, Dale must have gone $24 \times \frac{8}{2} = 96\, inches$

Second round Dale goes 24 more inches, so that is $24 + 96 = 120 \ \ inches$

OR

Once we know that every time Dale goes 24 inches, Mario goes 22 inches we can just keep track of how far they go and stop when Dale is ahead of Mario by 10 inches. To check our answer we check that if Dale is 24 inches behind the finish line, then Mario is 32 inches behind (this is where they are when Al wins).

Step	Dale (increase by 24 inches)	Mario (increase by 22 inches)	Is Dale ahead by 10 inches?
1	24	22	NO
2	48	44	NO
3	72	66	NO
4	96	88	NO
5	120	110	YES

Check: If the course is 120 inches long, then when Dale is 24 inches behind (step 4), then Dale is 120-88 = 32 inches behind so this checks!

2. Three adults and two kids want to cross a river by using a small canoe. The canoe can carry two kids or one adult. How many times must the canoe cross the river to get everyone to the other side?

Move	Adults	Kids	River	Adults	Kids	
1.	3	0	2K $\rightarrow$	0	0	
2.	3	0	K $\leftarrow$	0	1	
3.	2	1	1A $\rightarrow$	0	1	
4.	2	1	K $\leftarrow$	1	0	4 trips to get an adult across and boat back
5.	2	0	2K $\rightarrow$	1	0	
6.	2	0	K $\leftarrow$	1	1	
7.	1	1	1A $\rightarrow$	1	1	
8.	1	1	K $\leftarrow$	2	0	8 trips, 2 adults
9.	1	0	2K $\rightarrow$	2	0	
10.	1	0	K $\leftarrow$	2	1	
11.	0	1	1A $\rightarrow$	2	1	
12.	0	1	K $\leftarrow$	3	0	12 trips, 3 adults
13.	0	0	2K $\rightarrow$	3	0	
Result of trip 13				3	2	

3. Five logicians got together one evening to play some logic games. Their first game consisted of four players and one emcee who ran the game. The four players were Ryan, Torrey, Michael, and Bonnie. Janet acted as the emcee. Janet told the players that she was shuffling four white hats and three black hats and was going to put a hat on each of their heads. Each person would be given a chance to tell what color hat he or she was wearing. Janet told everyone to close their eyes. She then put a hat on each head and lined people up in single file, facing in the same direction. Ryan, the person in the back, could see all the other heads when he opened his eyes. Janet asked him if he knew what color hat he was wearing. He didn't know and said so. Torrey was next in line. She could not see Ryan, but could see the other two players. She also said she didn't know her hat color. Michael could see only Bonnie, and he said that he didn't know either. Bonnie, who was in front and could see no one else, knew the color of the hat on her head and announced it correctly. What color was her hat, and how did she know?

We will use the "eliminate possibilities strategy". Here are all the possibilities for the hat colors:

	Ryan (back of line)	Torrey	Michael	Bonnie (front of line)
1.	White	White	White	White
2.	Black	White	White	White
3.	White	Black	White	White
4.	White	White	Black	White
~~5.~~	~~White~~	~~White~~	~~White~~	~~Black~~
6.	Black	Black	White	White
7.	Black	White	Black	White
~~8.~~	~~Black~~	~~White~~	~~White~~	~~Black~~
9.	White	Black	Black	White
~~10.~~	~~White~~	~~Black~~	~~White~~	~~Black~~
~~11.~~	~~White~~	~~White~~	~~Black~~	~~Black~~
~~12.~~	~~White~~	~~Black~~	~~Black~~	~~Black~~
~~13.~~	~~Black~~	~~White~~	~~Black~~	~~Black~~
~~14.~~	~~Black~~	~~Black~~	~~White~~	~~Black~~
15.	Black	Black	Black	White

If the last person (Ryan) couldn't tell what color hat he was wearing, then it must be the case that not all 3 hats in front of him were black, since if they were then all 3 black hats were used and his must be white. This eliminates possibility 12.

If the second to last person (Torrey) saw 2 black hats, then she'd know that she was in case 11 or 13. In both of those cases her hat is white so she would know the color. Since she did not know the color we can eliminate 11 and 13.

If the next person (Michael) saw 1 black hat then he would know he was in either case 5,8,10, or 14. In any of these cases his hat would be white so he would know his color. Since he did not know his color, then we can eliminate possibilities 5,8,10 and 14.

Since no one else could tell what color their hat was, Bonnie knows that 1 – 4,6,7,9, and 15 are the only possibilities left. In all of these cases her hat is white so Bonnie knows her hat is white.

4. There are 48 people at a party. There are seven times as many adults as children. There are twice as many females as males. If there are 5 girls (female children) at the party, how many men (male adults) are there?

<u>Algebra</u>

MA = # of Male Adults FA = # of Female Adults MC = # of Male Children FC = # Female Children **Find MA = ?**

MA + FA + MC + FC = 48 (MA + FA) = 7(MC + FC) (FA + FC) = 2(MA + MC)

<u>One possible solution</u>:

FC = 5

MA + FA + MC + 5 = 48	(MA + FA) = 7(MC + 5)	(FA + 5) = 2(MA + MC)
MA + FA + MC = 43	(MA + FA) = 7(MC + 5)	(FA + 5) = 2(MA + MC)
(**MA + FA**) = 43 – MC	(**MA + FA**) = 7MC + 35	FA + 5 = 2MA + 2MC
43 – MC = 7MC + 35		FA = 2MA + 2MC – 5
8 = 8MC → MC = 1	→ →	FA = 2MA + 2(1) – 5 → FA = 2MA – 3

MA + (2MA – 3) = 43 – 1

3MA = 45 → **<u>MA = 15</u>**

OR

<u>Guess and check table (see next page)</u>

Female Children (5)	Male Children	Total Children	Total adults will be 7 x total children so I'm going to always choose the female and male adults to add up to this number	Total of kids and adults – do they add up to 48? Don't bother to go further if they don't	Female Adults	Male Adults	Total Female s	Total Males	Females = 2* Males?
5	5	10	70	80 – NO too many people, start over					
5	3	8	56	64 – NO still too many, start over					
5	1	6	42	48 – YES, let's keep going	30	42-30 =12	35	13	NO – too few males
5	1	6	42	48 – YES, try again	25	42-25=17	30	18	NO – too many males
5	1	6	42	48 – Yes, try again	27	42-27=15	32	16	YES!

Answer – there are 15 male adults at the party!

5. Al, George, Kathleen and Chad counted ballots in Florida. They were each given a box of ballots. Each box contained the same number of ballots. Each person counted ballots at a constant rate. However, the rate at which each person counted ballots was different. When Al finished counting all of his ballots, George had 54 ballots left to count, Kathleen had 90 ballots left to count and Chad had 106 ballots left to count. When George finished counting all his ballots, Kathleen had 45 ballots left to count and Chad had 65 ballots left to count. When Kathleen finished counting all of her ballots, how many ballots did Chad have left to count? How many ballots were in each box to start with?

Think of this group of people counting ballots in 4 rounds:
End of round 1(R1) – A finishes, G has 54 left, K has 90 left, C has 106 left
End of round 2(R2) – G finishes, K has 45 left, C has 65 left
End of round 3(R3) – K finishes, C has ?? left

R1 to R2: K counts 45 ballots (90 – 45 = 45), C counts 41 ballots (106 – 65 = 41)
R2 to R3: K counts 45 ballots so then C must have counted 41 ballots. So then C has 65 – 41 = 24 Ballots

1st answer: C has 24 ballots left to count

End of R1 G has 54 and K has 90, so G has counted 36 more than K at end of R1.
R1 to R2 we know that G counted 54 while K counted 45 for a difference of 9. So for each group of 45 that K counts, G will count 9 more.

Work backwards: At the end of R2 the difference between G and K was 45, so how many rounds did it take for K to fall behind G by 45? Remember G does 9 more than K each round, so 45 ÷ 9 = 5. So it must have taken 5 rounds for G to finish and K to still have 45.
Thus 5 rounds of G counting at 54 ballots per round is 5 x 54 = 270 total ballots.

2nd answer: 270 total ballots for each person.

6. Abbie, Bridget, Cynthia and Dena are women whose professions are water quality engineer, soil contamination scientist, air pollution consultant and biological diversity advocate. Match each woman to her expertise, using the follow clues.
    1. Bridget and the biological diversity advocate are both from Oregon.
    2. Abbie, the soil contamination scientist and the air pollution consultant all love to garden.
    3. The air pollution expert, the water quality engineer and Dena all met one another at a global warming conference.
    4. Bridget has never met the person who works on air pollution.

	Abbie	Bridget	Cynthia	Dena
H_2O Engineer	Yes	$X_{3\&4}$	X	X_3
Soil scientist	X_2	Yes	X	X
Air consultant	X_2	X_4	Yes	X_3
Bio advocate	X	X_1	X	Yes

Abbie is H_2O Engineer
Bridget is Soil scientist
Cynthia is Air consultant
Dena is Bio advocate

7. Trivia took some fishbowls she'd bought to the flea market. In the first hour, she sold one-third of them and a third of one more. In the second hour, she sold half of them, and half of one more. In her third hour there, she sold one-third of them and a third of one more. The next hour, she sold half of them and half of one more. Finally, she sold the last two and went home. How many fishbowls did Trivia sell?

   Strategy: work backwards.

I am going to represent this with a diagram:

SOLD 1/3 of total		
1st hour	1/3 more	
	sold ½ of remaining	
2nd hour	½ more	
	sold 1/3 of remaining	
3rd hour	1/3 more	
	sold ½ of remaining	
4th hour	½ more	
5th hour	sold last 2	

Now we work from the bottom and fill in as we go up: Total = 52/6 x 3 = 26 fishbowls

SOLD 1/3 of total	52/6	52/6	
1st hour	1/3 more	17	
	sold ½ of remaining	8.5	
2nd hour	½ more	8	
	sold 1/3 of remaining	16/6	16/6
3rd hour	1/3 more	5	
	sold ½ of remaining	2.5	
4th hour	½ more	2	
5th hour	sold last 2		

Check: Start with 26
1st hour: sell 1/3 of 26 + 1/3 more is 26/3 + 1/3 = 27/3 = 9
remaining 26-9 = 17
2nd hour: sell 1/2 of 17 + ½ more is 17/2 + ½ = 9
remaining 17-9 = 8
3rd hour: sell 1/3 of 8 plus 1/3 more is 8/3 + 1/3 = 9/3 = 3
remaining 8-3 = 5
4th hour: sell ½ of 5 plus ½ more is 5/2 + ½ = 6/2 = 3
remaining 5-3 = 2
5th hour: sell remaining 2
2-2=0 that’s it!